Review of Parameter Estimation Techniques for Time-Varying Autoregressive Models of Biomedical Signals

A. R. Najeeb, M. J. E. Salami, T. Gunawan, and A. M. Aibinu
Department of Electrical and Computer Engineering, International Islamic University of Malaysia, Kuala Lumpur, Malaysia
Email: {athaur, momoh, teddy}@iium.edu.my, aibinu@gmail.com

***Abstract*—Biomedical signals are non-stationary and a research topic of practical interest as the signal has time varying statistics. The problem of time varying is usually circumvented by assuming local stationary over a short time interval, where stationary techniques are applied. However, features extracted from these methods are not always suitable and methods for non-stationary process are needed. Time varying signals are more accurately represented by time frequency methods and received most attention recently. Among the time frequency methods, parametric modeling such as TVAR has been promising over non-parametric methods with improved resolutions and able to trace strong non-stationary signal. Despite the success of TVAR in various applications it has drawbacks. This paper presents an extensive review on TVAR modelling techniques. Different approaches for TVAR modeling is presented and outlined. Principles, advantages, disadvantages of those techniques are presented concisely. And finally a new direction has been suggested briefly.**

***Index Terms*—autoregressive spectral analysis, biomedical signal processing, model order determination, non-stationary signal analysis time varying coefficients, genetic algorithm, artificial neural network**

I. Introduction

A common routine for dealing with non-stationary signal is to partition into several segments (window) of known length; after which traditional methods of nom-parametric methods or parametric method such as Fast Fourier Transform (FFT) or Autoregressive (AR) is applied [1]. The signal is assumed to be stationary within this short-duration window; however, this method may not work well if averaging of Power Spectral Density (PSD) from different segments fails to capture the dynamics of the data [2]. Furthermore, each segment is assumed to be independent but for many signals, this is not the case as each segment is statistically depends on the next. While some data turn out to be too short, leading to that the estimates become unreliable due to few data points and cannot partitioned into several segments. This has led to a growing interest in non-stationary signal processing including Time-Frequency Representation (TFR) techniques which able to capture changing dynamics of both deterministic and random signal. TFR also works well with short data. Available TFR methods are categorized into non-parametric methods and parametric methods as shown in Fig. 1.

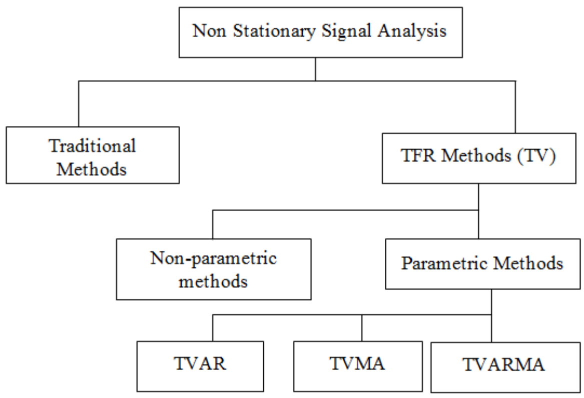

Figure 1. Classification of non-stationary signal analysis

The most extensively exploited non-parametric method includes Short Term Fourier Transform (STFT), Smoothed Pseudo Wigner-Ville (SPWV), Wavelet Transforms, Gabor Transform (GT), Hilbert Transform (HT), Continuous Mortlet Wavelet Transform (CMWT), Wigner-Viler (WVD) and their enhanced derivations. These methods are computational efficient and do not make any assumptions about the process except for its stationarity, which makes them as methodology of choice particularly in situations where long data need to be analyzed. A good summary of properties, mathematical model and application of these techniques can be found in [2]-[5].

Despite their success, there are some drawbacks of these methods, for example, the window effects, the low time-frequency resolution in STFT and the cross-term interference in WD. STFT is essentially composed of piece-wise FFT, which assumes that the signal is locally stationary in each segment The segment size is so critical to performance as there exist a trade-off between time resolution and frequency resolution in accordance with uncertainty principle [6]. Choosing a short segment size causes poor frequency resolution while a long segment size compromises the assumption of stationary data [7].

Manuscript received February 10, 2015; revised July 13, 2015.

doi: 10.18178/ijsps.4.3.220-225

Although the alternate methods such as WVD yields good resolutions in both time and frequency for systems with single components, however when applied on multi-component signals they produce a lot of artifacts [8]. Consequently, non-parametric methods are limited by applications and hence are not suitable for broad range of applications.

The best frequency resolution for non-stationary signal is obtained by using parametric models where the signal is fitted into an autoregressive (AR), moving average (MA) or an autoregressive moving average (ARMA) model. More parsimonious representation of signals and higher resolution of time-frequency spectra are achievable even for a small length of non-stationary signal using these models. Moreover, the parametric approaches are able to track relatively fast TV dynamics and detect multiple TV spectral peaks which may not be achieved by the non-parametric methods [9]. Un-availability of long data in biomedical applications such as EEG, ECG or tumor analysis certainly leads to parametric methods as preferred method. [10]

Time varying Autoregressive (TVARAR) models have been investigated by many researchers and received the most attention in literatues among theexisting parameteric modeling techniques in last few years [11]. This is popular assumption for several reasons such as: 1) many natural signals has underlying autoregressive structure, 2) any non-stationary signal can be modeled as a AR process if sufficient model order is selected, 3) estimation of AR model parameters involves linear system of equations which can be solved efficiently, 4) the computational load to calculate the AR model parameters tend to be less than that for MA or ARMA models.

Despite the success of TVAR in varous applications, it has few drawback, namely the accuracy of TVAR coefficient estimation algorithm and the complexity associated with determining the optimum model order. Incorrect parametersoften leands to introduction to artifacts, spurious spectral peaks, false valleys which will lead to unstable system and consequently this method will fail. In this paper, methods available to estimate TVAR parameters are presented and commented.

In next section, we present the TVAR model with its basic equation. Different TVAR parameter estimation methods are presented and their computational aspects are addressed. In Section 3, we compare and discuss strength and limitation of those algorithms and finally the conclusion in Section 5.

II. TVAR Coefficient Estimation Methods

A TVAR process which is driven by a white noise sequence can be expressed as:

$$y[n] = -\sum_{j=1}^{p} a_j[n] y[n-j] + \sigma\ [n] \tag{1}$$

where:

$a_j[n]$; j=1, 2, 3…p are time varying AR coefficients

p is the model order

$\sigma[n]$ is zero mean, stationary Gaussian white noise

To model a signal using TVAR parameters, p and $a_j[n]$ is computed. Although p determines the accuracy of TVAR, many researchers assumes it is known and estimating only $a_j[n]$.

As TVAR coefficient is now a time varying parameter, popular TIVAR methods developed as Levisohn-Durbin algorithm or Burg algorithm may not produce desirable results.

Methods for TVAR coefficient estimation can be categorized into three classes: adaptive recursive estimation methods, deterministic basis function expansion method or a hybrid method. Their classification is dissipated in Fig. 2. Background of these categories is revised and commented further in this section.

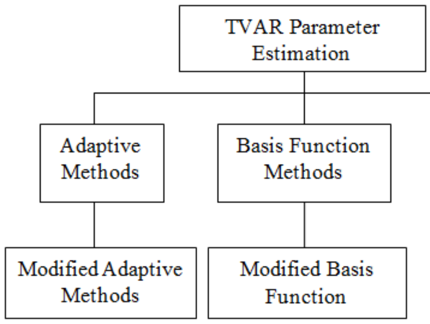

Figure 2. Classification TVAR parameter estimation techniques

A. *Adaptive Methods (AM)*

Adaptive TVAR are among the ealiest methods which practically useful in many biomedical signal processing applications. Many variation of adaptive algorithms were studied, but the most popular ones are Least Mean Square (LMS), Recursive Least Square (RLS) and Kalman Filtering (KF). Details about these algorithm and underlying mathematics are available in [9]-[12]. In these methods, variation of $\mathrm{a_j[n]}$ are based on a dynamic model which is defined as:

$$a_j[n] = a_j[n-1] + \Delta a_j[n] \tag{2}$$

where, $a_j[n]$ are updated from their previous values of $a_j[n\text{-}1]$.

$\Delta a_j[n]$ represents an innovation terms which depends on type of adaptive algorithm used. For example in LMS, the $\Delta a_j[n\ ]$ is equal to $\mu \cdot \mathrm{E}\{\mathrm{e[n]}\overline{\varphi}(\mathrm{n})$ or $\cdot\, \mathrm{e(n)}\, \overline{\varphi}(\mathrm{n})$, respectively in which $\overline{\varphi}(\mathrm{n}) = [\mathrm{x(n)}\ \mathrm{x(n-1)} \cdots \mathrm{x(n-p)}]^{\mathrm{T}}$ is a vector of the sampled nonstationary signal.

If RLS is applied, an additional parameter known as forgetting factor is introduced. When RLS compared with LMS, LMS offers faster and easy implemention method whereby it can be applied with limited knowledge on input signals. The RLS which has more complex structure, exhibits better performance and fewer iterations. LMS and RLS are sufficient for TVAR parameter estimation if the non-stationary part of the signal changes slowly and not suitable for rapidly changing dynamics [13]. Although these methods work reasonably well for slow varying signals but they are also sensitive to noise. The noice sensitivity may be reduced by increasing the step size or forgetting factor, but the convergence rate will be decreased as well and this result in a diminished ability in tracking the parameter change. [14] To further enhance the stability, these parameters are defined within a range determined by largest eigenvalue; which is an assumed value. [13]

B. Modified Adaptive Methods (MAM)

The Normalized LMS (NLMS), Weighted Size LMS, Modified Block LMS, Variable Step Size LMS (VSS-LMS), Variable Forgeting Factor (VFF), state-based VFF and Modified VSS-LMS are among studied algorithms which has shown to increase the convergense speed. Their mathematics are well presented consicely in. [15]

Although the modified LMS has shown to increase the covergence speed, however, the performance of these algorithms is sensitive to the selection of step sizes with more coefficients with increase in computation steps. In some cases modified LMS algorithms has serious signal distortion when applied to biomedical applications. This makes the modified adaptive method less studied when algorithm for broad application is attempted.

Modified RLS such as T-RLS, S-RLS methods has been a subject of research as well, but its application on biomedical signals are less popular as the nature of RLS is complex; therefore, their modificaton inherits the those complexity as well. Furthermore, they do not not produce significantlly better MSE when compared to modified LMS.

C. Basis Function Methods (BFM)

The BFM is a deterministic parametric modelling approach, where the $a_j[n]$ are expanded as a finite sequence of pre-determined basis function :

$$a_j[n] = \sum_{i=0}^{m} a_{ji} f_i[n] \tag{3}$$

where $a_{ji}, j=1, 2, \cdots, p$, $i=0, 1, \cdots m$ are constants. m is expanson dimensions and $f_i[n]$ is the predefined basis function. Therefor, (1) can be re-wriiten as:

$$y[n] = -\sum_{j=1}^{p}\left(\sum_{i=0}^{m} a_{ji}[n] f_i[n]\right) y[n-j] + \sigma\ [n] \tag{4}$$

With an estimation error of

$$e[n] = \hat{y}[n] - y[n] \tag{5}$$

To estimate $\mathrm{a_j}[\mathrm{n}]$ from (4), parameters p, $a_{ji}[n]$ and m is to be calculated recursively to reach an optimized value which will give minimum error for a selected value of $f_i[n]$. Therefore, the process become iterative and long, leading to a slow and complex computation with total number of $(m + 1)$ x p parameters estimated.

It is a common practise among researchers to fix the orders, p and m to reduce the computation burden [16]. Or other approach is to test for different set of p and m manually to select the best value, before applying $a_{ji}[n]$ estimation algorithm.

Among earliest work in determining $a_{ji}[n]$ is by Hall et al with a modified Linear Predictive Coding approach [16]. In this approach $\cos(i\omega n)$ and $\sin(i\omega n)$ is selected to form the $f_i[n]$. A generalized correlation function, $c_{kl}(i,j)$ is defined between basis function and data sequence. Then (4) is rearranged in matrix of c-terms where a least square error technique is used to determine a_{ji}. Coefficients is optimized by minimizing the total square error, $E = \sum_n e^2(n)$. However, method for model orders are not studied in this work. Details on this algorithm is available at [16], [17].

A similar approach was studied by [18] recently in 2014, adopting a dynamic approach with three major changes. First, instead of correlation, a covariance relationship were applied. Secondly, basis function is formed by two tradional polynomial function namely Legendre and Chebyshev instead of the trigonomic functions. And lastly, in earlier work, a constant model orders for *p*, *m* of (2, 2) were used while in the later work, the reseacher has adopted a dynamic computation of model orders by means of maximizing the likehood function, MLE.

Another classical work was produced by [19], where development of a novel Bayesian formulation to determine the model order. The model were let to be over-modeled and later decomposed using a method known as Discrete Karhunen-Loeve Transform (DKLT), in order to align the AR coefficients along a direction towards greatest energy. Later, smoothed by applying SVD to produce a orthoginal basis set of AR coefficients.

Although large number of coefficients are to be determined, the BFM has superior performances over AM. Where, it is able to trace a strong non-stationary signals, able to detect multiple time-varying peaks in the presence of noice, yields more information for spectral analysis, improved resolution in both domain, suitable for short data and able to detect rapidly varying signals. [3], [9], [15], [16], [20]-[23]

Despite of their success, there are two major drawbacks of BFM. Firstly, to select significant basis function from the pool of available basis functions. Numerous basis functions are projected in literatures such as Time Basis functions, Fourier Basis, Walsh and Haar functions, Multiwavelet, Discrete Prolate Spheroidal Sequences, Chebyshev Polynomial, Legendre Polynomial, Diecrete Cosine Functions but however there is no specific guideline on selection of appropiate basis functions. In fact, a single set of basis function has its own unique characteristics can best capture dynamics of the system with similar features. Therefore the use of a single set of basis function is inadequate for biomedical signals as these signals compose of both fast and slow varying signals.

Second issue with BFM approach is the accuracy of model orders, p and m. The accuracy of estimated $a_j[n]$ is sensitive to the choise of model order. The model order determines the amount of memory required to present the process. If the model order is inappropiate, the model parameters will not characterize the underlying nature of process and will not represent the signal. From spectral analysis perspective, low model order will produce smoothed spectral and a high model order will cause supirious spectral peaks. Furthermore, the present model order determination techniques such as Akaike Information Criterion, Bayesian approach are designed for conventional AR process such unfit for a TVAR process. [24]

D. Modified Basis Function Methods (MBFM)

MBFM methods were introduced to overcome the limitations of BFM which includes dynamic computation of model orders or to study on optimized basis functions. Optimal Parameter Search Algorithm was proposed in [24] where the authors accurately determine the model order and extract the significant model terms by discriminating irrelevant basis sequence. It has been shown that this method works well for overparametrized, corrupted signals and is also applicable for linear and non-linear system. Then irrelvant basis sequence where removed pool of candidate vectors and a liner independent matrix were formed before least square method is applied. A projecton distance is calculated to compute model orders p and m. Although this proved better performances, but more parameters were introduced and it becomes increasingly complex at each level of n. The same methods were studied again using multiple set of basis function [12] with similar success but again with huge number of coeffiecients and intermediate parameters were used.

Accuracy of model order may also increased is by adopting forward and backward (FB) TVAR. A system with such approach is flexible and has superior performance over the model using only causal (forward) TVAR as in (1). An anti-causal or backward TVAR is defined as:

$$\hat{y}^b[n] = -\sum_{j=1}^{p} b_j[n]y[n+j] \quad (6)$$

A quick look on the forward and backward equations may suggest the $b_j[n]$ and $a_j[n]$ need to be computed independently, and thus computation time will be doubled. But, as the matter of fact, it is not, as $b_j \cong a_j^*$ is assumed.

[25] proposed a FB TVAR scheme with a time delay. With $a_j[n]$ and $b_j[n]$ is computed using methods proposed by [16] and consequently a modified MSE used as optimization criteria. In this method, a double computation for the symmetric matric, **C** is unavoidable. These method were shown effective in In a noisy enviroment and high model order While, [18] proposed a Modified Covariance Method to form a Blok Matrix, C and applied Wax-Kailath Algorithm to solve for $Ca = -d$ and hence the algorithm becomes more complex by computing model order dynamically.

E. Hybrid Methods (HM)

A HM were proposed by [26] where TVAR process is approached with a novel multi-wavelet decomposition scheme consisting sum of multiple set of wavelets family. By this definition the TVAR is now reduced to regression selection problem. Parameters are then resolved by using Forward Orthogonal Regression Algorithm.

Different HM proposed by [27] and [28] where basis function defined by multi wavelet decomposition and modified block LMS were employed to estimate $a_j[n]$. Different signals consisting fast and slow changing dynamics, solves the inherit limitation of LMS. [29] used the multi wavelet expansion to represent TVAR and a normalized LMS to estimate the parameters; and produces similar conculsion to previous research.

Therefore the adaptive method when used with a multi-wavelet basis function expansion has proven to overcome their inheritant limitations. However this approaches involves a great number of candidate model terms and increases the computation time. If dynamic computation of model order is employed, the computation becomes much heavier as this approaches are in direct form.

III. Discussion and Recommendation

In recent past, new research field in Medical Informatics have emerged while current technologies were updated in short span of time. Design and development of medical instruments are becoming increasingly complex to meet the needs of human endeavors, and such the data formats become more complex. This leads to demand for more sophisticated biomedical signal processing algorithms which can be implemented in broad range of applications.

As the traditional methods no longer meet the demands of current technology the time variant methods of signal analysis are increasingly becoming the preferred method for processing of biomedical signals. Among the various TFR methods, parametric method characterized by AR transfer function is studied well by researcher as the TVAR provides more details on spectrum data in comparison with non-parametric techniques. However, the success of TVAR is determined by the accuracy of model orders (p, m) and algorithm to estimate the TV coefficients.

Available TVAR coefficient computing algorithms falls in two broad categories. Details of these algorithms have been discussed in Section 2.0. Their strengths and limitations are further summarized in Table I. Table I is not representation of a performance analysis on these methods, but rather comparison of their strength and limitation. In each of these categories, different algorithms have been researched and shown to working well in their scope of application.

From Table I, we could conclude that the TVAR model identification via BFM has shown advantages and better performances over AM. It is reliable in detecting signals with multi-dynamics and with multiple peaks as well. However, the selection of model orders, expansion dimensions and type of basis function is of concern since there is no fundamental theorem on how to choose them.

Furthermore, implementation of BFM and MBFM is currently via direct approach where optimization is reached by performing recursive and iterative computations as per the mathematic model. Although the results were promising; however they are computationally expensive with huge number of coefficients to be defined.

TABLE I. COMPARISON OF TVAR PARAMETER ESTIMATION ALGORITHMS

Categories	Strength	Limitations
Adaptive Algorithm	• Easy Implementation, • Low Cost	• Detect slow changing dynamics only • Limited by uncertainty principle
Modified Adaptive Algorithm	• Detects variety of signals	• Complex implementation • High Computation • Slow Convergence Rate • Application dependent • Iterative and recursive • Large number of coefficients
Basis Function Algorithm	• Detects variety of signal • Higher resolution in both time domain and frequency domain • Not subjected to uncertainty principle	• Model order dependent • Basis Function dependent • High number of coefficients • High Computation • Recursive and Iterative
Modified Basis Function Algorithm	• Broader Applications • Detects variety of Signals • Higher resolution in both time domain and frequency domain • Not subjected to uncertainty principle • Noise reduction in noisy signal • Adjustment of over parameterized model order	• Model Order dependent • High number of coefficients High • Computation Recursive and Iterative
Hybrid Techniques	• Inherits advantages of basis function and modified adaptive methods	• High number of coefficients • Recursive and Iterative • High computation

From the categories of algorithms shown in Table I, Modified Basis Function (MBF) should be considered for further improvement. Under MBF two schemes have been proposed where first, a combination of different set of basis function and second one is a forward-backward TVAR. Combination of basis function addresses the question of which basis function should be employed to detect different types of signals. Combining few basis functions also could trace changing sharp dynamics in analyzed signal. As such further studies should consider this fact. Adopting forward-backward TVAR will reduce over fitted model orders and to filter noise as well.

Recursive computation of BFM can be reduced by employing intelligent Artificial Neural Network (ANN). The superior performance of ANN has been studied demonstrated on AR model identification of stationary signals [30]. The use of Genetic Algorithm is used accurately determine the model order in non-stationary cases as well. As such this combination can be extended into non-stationary signals as well.

However to our knowledge in non-stationary biomedical signals analysis, no work has been done to exploit the superior performance of GA and ANN. Therefore an intelligent BFM algorithm based on GA and ANN is highly recommended as the way forward to further enhance the performance and to reduce the computation complex in current BFM methods.

IV. CONCLUSIONS

As a conclusion, we had reviewed TVAR parameter estimation methods in relation to biomedical signals. Their strengths and limitations are summarized in Table I. BFM has been identified as promising approach; however it has recursive computation which increases the number of parameters to be estimated before optimized parameters are obtained. To further enhance the performance of BFM, it is proposed to employ ANN and GA algorithm into BFM, which will precisely estimate the parameters with reduced computation time. BFM adopting both ANN and GA for biomedical signals is yet to be explored. Multiple basis function will further improve the algorithm to trace multiple dynamics of non-stationary data. It is our hope that our work will provide some useful insights and perspective for future work on time-varying signal representations.

REFERENCES

[1] M. Hu and H. Shao, "Autoregressive spectral analysis based on statistical autocorrelation," *Physica A Statistical Mechanics and Its Applications*, vol. 376, no. 1, pp. 139-146, 2007.

[2] J. Muthuswamy, "Biomedical signal analysis," in *Standard Handbook of Biomedical Engineering and Design*, Tempe: McGraw-Hill Companies, 2004, pp. 1-30.

[3] H. L. Wei, J. Liu, S. A. Billings, and M. Street, "Time-Varying parametric modelling and time-dependent spectral characterisation with applications to EEG signals using multi-wavelets," *Int. J. Model. Identif. Control*, vol. 9, no. 3, pp. 215-224, 2010.

[4] F. Hlawatsch and G. F. B. Batels, "Linear and quadtratic time frequency signal representations," *Signal Process. Mag.*, vol. 9, no. 2, pp. 21-67, 1992.

[5] E. N. Bruce, *Biomedical Signal Processing and Signal Modeling*, Canada: John Wiley and Sons, 2001.

[6] M. Wacker and H. Witte, "Time-Frequency techniques in biomedical signal analysis: A tutorial review of similarities and differences," *Methods Inf. Med.*, vol. 52, no. 4, pp. 279-296, Jan. 2013.

[7] Y. Wang, S. Ji, and H. T. Xu, "Non-Stationary signals processing based on STFT," in *Proc. 8th Int. Conf. Electron. Meas. Instruments*, Aug. 2007, pp. 3-301-3-304.

[8] L. Zhang, G. Xiong, H. Liu, H. Zou, and W. Guo, "Time-Frequency representation based on time-varying autoregressive model with applications to non-stationary rotor vibration analysis," *Sadhana*, vol. 35, no. 2, pp. 215-232, Apr. 2010.

[9] K. B. Eom, "Analysis of acoustic signatures from moving vehicles using time-varying autoregressive models," *Multidimens. Syst. Signal Process.*, vol. 10, pp. 357-378, 1999.

[10] L. Faes, K. H. Chon, and G. Nollo, "A method for the time-varying nonlinear prediction of complex nonstationary biomedical signals," *IEEE Trans. Biomed. Eng.*, vol. 56, no. 2, pp. 205-209, Feb. 2009.

[11] A. M. Aibinu, M. J. E. Salami, and A. A. Shafie, "Artificial neural network based autoregressive modeling technique with application in voice activity detection," *Eng. Appl. Artif. Intell.*, vol. 25, no. 6, pp. 1265-1276, Sep. 2012.

[12] R. Zou and K. H. Chon, "Robust algorithm for estimation of time-varying transfer functions," *IEEE Trans. Biomed. Eng.*, vol. 51, no. 2, pp. 219-228, Feb. 2004.

[13] S. Elouaham, R. Latif, and A. Dliou, “Parametric and non parametric time-frequency analysis of biomedical signals,” *Int. J. Adv. Comput. Sci. Appl.*, vol. 4, no. 1, pp. 74-79, 2013.
[14] F. Shiman, S. H. Safavi, F. M. Vaneghi, M. Oladazimi, M. J. Safari, and F. Ibrahim, “EEG feature extraction using parametric and non-parametric models,” in *Proc. IEEE-EMBS International Conference on Biomedical and Health Informatics*, Hong Kong, 2012, vol. 25, pp. 66-70.
[15] E. Eweda, “Advances in nonstationary adaptive signal processing,” in *Proc. Thirteenth National Radio Science Conference*, Cairo, 1996, pp. 61-82.
[16] M. G. Hall, A. V. Oppenheim, and A. S. Willsky, “Time-Varying parametric modeling of speech,” *Signal Processing*, vol. 5, no. 049, pp. 267-285, 1983.
[17] G. R. S. Reddy, R. Rao, and C. V. R. College, “Performance analysis of basis functions in TVAR model,” *Int. J. Signal Process. Image Process. Pattern Recognit.*, vol. 7, no. 3, pp. 317-338, 2014.
[18] G. R. S. Reddy and R. Rao, “Instantaneous frequency estimation based on time-varying auto regressive model and WAX-kailath algorithm,” *An Int. J. Signal Process.*, vol. 8, no. 4, pp. 43-66, 2014.
[19] J. J. Rajan and P. J. W. Rayner, “Generalize feature extraction for time varying autoregressive models,” *IEEE Trans. on Signal Processing*, vol. 44, no. 10, Oct. 1996.
[20] M. J. Cassidy and W. D. Penny, “Bayesian nonstationary autoregressive models for biomedical signal analysis,” *IEEE Trans. Biomed. Eng.*, vol. 49, no. 10, pp. 1142-1152, Oct. 2002.
[21] Y. Li, H. Wei, S. A. Billings, P. Sarrigiannis, S. A. Billings, and P. G. Sarrigiannis, “Time-Varying model identification for time-frequency feature extraction from EEG data,” *J. Neurosci. Methods*, vol. 196, no. 1021, pp. 1-13, 2011.
[22] R. Zou, H. Wang, and K. H. Chon, “A robust time-varying identification algorithm using basis functions,” *Ann. Biomed. Eng.*, vol. 31, no. 7, pp. 840-853, Jul. 2003.
[23] G. R. S. Reddy, R. Rao, and C. V. R. College, “Non-Stationary signal analysis using TVAR model,” *Int. J. Signal Process. Image Process. Pattern Recognit.*, vol. 7, no. 2, pp. 411-430, 2014.
[24] A. H. Costa and S. Hengstler, “Adaptive time-frequency analysis based on autoregressive modeling,” *Signal Processing*, vol. 91, no. 4, pp. 740-749, Apr. 2011.
[25] C. Sodsri, “Time-Varying autoregressive modelling for nonstationary acoustic signal and its frequency analysis,” Ph.D. thesis, The Pennsylavania State University, Dec. 2003.
[26] K. H. Chon, H. Zhao, R. Zou, and K. Ju, “Multiple time-varying dynamic analysis using multiple sets of basis functions,” *IEEE Trans. Biomed. Eng.*, vol. 52, no. 5, pp. 956-960, May 2005.
[27] Y. Li, H. L. Wei, S. A. Billings, and H. Wei, “Time-Varying signal processing using multi-wavelet basis functions and a modified block least mean square algorithm,” Research Report, ACSE Research Report No. 998, Automatic Control and Systems Engineering, University of Sheffield, pp. 1-17, 2009.
[28] Y. Li, H. Wei, and S. A. Billings, “Identification of time-varying systems using multi-wavelet basis functions,” *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 3, pp. 656-663, May 2011.
[29] S. N. Mate and A. B. Patil, “Identification of time varying system using recursive estimation approach and wavelet based recursive estimation approach,” *Int. J. Eng. Sci. Innov. Technol.*, vol. 2, no. 3, pp. 526-532, 2013.
[30] J. Tian, M. Juhola, and T. Grönfors, “AR parameter estimation by a feedback neural network,” *Comput. Stat. Data Anal.*, vol. 25, no. 1, pp. 17-24, Jul. 1997.

Athaur Rahman Bin Najeeb is a PhD student in Faculty of Engineering, Department of Electrical and Computer Engineering. He carries his research under supervision of Prof. Dr. Momoh Jimoh E. Salami and Dr. Teddy Gunawan. Athaur Rahman Bin Najeeb received his Master of Science in Electrical and Computer Engineering from IIUM in 2008. His research interests focus on Biomedical Signals Processing, Image Processing.

Prof. Dr. Momoh Jimoh Eyiomika Salami is Professor of Signal and Image Processing, International Islamic University Malaysia Professor at the Department of Mechatronics. He has authored/co-authored more than 100 publications in both local and international journals and conference proceedings as well as being one of the contributors. He had contributed a chapter in recently published book entitled “The Mechatronics Handbook” edited by Prof. Bishop. His research interests include Digital Signal and Image Processing, Intelligent Control Systems Design and instrumentation. Prof. Momoh is a senior member of IEEE.

Dr. Teddy Gunawan obtained his M. Eng from School of Computer Engineering, Nanyang Technological University in 1999 and his PhD in Electrical Engineering from School of Electrical Engineering and Telecommunications He is a Senior Member of IEEE. His research interests are Speech and audio signal processing, Genomic signal processing, Biomedical instrumentation and signal processing, Image processing and computer

Dr. Abiodun Musa Aibinu is Associate Professor and a technical expert in emerging application of Signal Processing. He obtained his PhD in Signal Processing from IIUM in 2011. Currently he is Director, at Center for Open Distance and e-Learning (CODEL), FUT Minna. His research interests include: Biomedical Signal, Processing; Digital Image Processing; Instrumentation and Measurements; Telecommunication system Design and Digital System Design.