NONLINEARITY IN RADIO REFRACTIVITY OVER AKURE, SOUTHWESTERN NIGERIA[1]

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Abstract
Chaos has found applications in many disciplines including radio communication. It has been applied to secure communication and time series analysis of many parameters. This study investigates chaos in the calculated refractive index over Akure using a two year data obtained from ongoing measurement of some atmospheric parameters by the communication and atmospheric physics research group, FUT, Akure. Chaos in the time series was investigated using phase space reconstruction, time delay, embedding dimension and Lyapunov exponent. Preliminary results shows positive Lyapunov exponent, hence, long term prediction of refractive index is impossible.

Keywords: Chaos, refractive index, radio communication, tropical location, Lyapunov exponent

INTRODUCTION

Chaos, as a very interesting nonlinear phenomenon, has been intensively studied over the past decades. It is defined as the aperiodic long-term behaviour in a deterministic system that exhibits sensitive dependence on initial conditions (Strogatz, 1990). Dynamic chaos has aroused considerable interest in many areas of science and technology due to its powerful applications in chemical reactions, power converters, biological systems (Njah and Ojo, 2010), economics (Kyrtsou and Labys, 2006), finance (Yao and Lui , 2010), secure communication (Ojo and Ogunjo, 2012), cryptography (Teh-lu and Shin-Hwa, 2000).

Since the discovery of a dynamical system that describe weather, (Lorenz, 1963) many dynamical systems have been proposed notably; Chua, Logistic, Rossler, Duffing, Van der Pol etc.. It has also been extended to systems called hyperchaotic such as Rabinovich, Lorenz, Rossler, among others. However, many real life systems generate data that are continuous in time, hence, the need to extend the study of chaos to time series data. Chaos

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theory has also been investigated in a wide range of time series data from physical experiments including short-term movements in asset returns (Hsieh, 1993), earthquake (Sarlis and Christopoulos, 2012; Kortas, 2005), geomagnetic horizontal field **(**George et al., 2001), geomagnetic pulsation (Vorda et al.,1994), menstrual cycle in women (Derry and Derry, 2010), stock market (Brock et al., 1991), sunspot data (Mundt et al., 1991), rainfall (Sharafi et al., 1990), Electrocardiogram (ECG) and electroencephalogram (EEG) (Zhang et al., 1992, Rapp et al., 1985), solar wind flow (Shollykutty and Kurian, 2009), traffic management (Li and Gao, 2004) among others.

Radio refractive index is an important parameter in determining the quality of UHF, VHF, and SHF signals. In characterizing a radio channel, surface (ground level) and elevated refractivity data are often required; and in particular, the surface refractivity is very useful for prediction of some propagation effects. Local coverage and statistics of refractivity, such as refractivity gradient, provide the most useful indication of the likely occurrence of refractivity related effects required for prediction methods (Bean and Dutton, 1968).

Consequences of refractivity are refractivity gradient and k-factor. Numerous works have been done to calculate the refractive index or any of its consequences: These include (Adediji et al, 2010), radio refractive index (Falodun and Ajewole, 2006) refractivity gradient over Botswana (Afullo et al, 1999), Adediji and Ajewole (2008) and so on.

This work investigates the nonlinearity (chaos) in the time series of refractive index over Akure using Lyapunov exponent as a quantifier. Lyapunov exponent is an important quantifier in the field of chaos as a positive exponent indicate nonlinearity. The significance of this is the impossibility of long term prediction of the time series. Time delay and embedding dimension were used in drawing the phase space and calculating the Lyapunov exponent.

METHODOLOGY

The concept of phase space representation rather than a time or frequency domain approach is the hallmark of nonlinear dynamical time series analysis. To recreate the phase space of the time series $x(t_0), x(t_1),..., x(t_i),..., x(t_n)$, we extend to a phase type of m dimensions phase space with time delay tau.

$$\begin{pmatrix} x(t_0) & x(t_1) & x(t_i) & x(t_n+(m-1)\tau) \\ x(t_0+\tau) & x(t_1+\tau) & x(t_i+\tau) & x(t_n+(m-2)\tau) \\ \vdots & \vdots & \vdots & \vdots \\ x(t_0+m(m-1)\tau & x(t_1+(m-1)\tau) & x(t_i+(m-1)\tau) & x(t_n) \end{pmatrix} \quad (1)$$

A phase point of phase space is made up of every row in equation (1). Every phase points has m weights and embodies a certain instantaneous state and the point's trajectory of phase space is composed of the link-line of phase point. Thus, the system dynamics can be studied in more dimensions phase space.

There is need to estimate an optimal value of time delay and embedding dimension to obtain good representation of phase space. Time delay can be evaluated using either of autocorrelation function or mutual information (Fraser and Swinney, 1986).

The average mutual information is defined as

$$I(\tau) = \sum_{X(i),X(i+\tau)} P(X(i), X(i+\tau))\, log_2 \left[\frac{P(X(i), X(i+\tau))}{P(X(i))\, P(X(i+\tau))} \right] \quad (2)$$

where i is total number of samples. $P(X(i))$ and $P(X(i+\tau))$ are individual probabilities for the measurements of $X(i)$ and $X(i+\tau)$. $P(X(i), X(i+\tau))$ is the joint probability density for measurements $P(X(i))$ and $P(X(i+\tau))$. The appropriate time delay, τ, is defined as the first minimum of the average mutual information $I(\tau)$. Then the values of $X(i)$ and $X(i+\tau)$ are independent enough of each other to be useful as coordinates in a time delay vector but not so independent as to have no connection with each other at all.

The sample autocorrelation of a scalar time series x(t) of N measurements is as presented in equation (3)

$$\rho(T) = \frac{\sum_{n=1}^{N}(x_{n+T} - \hat{y})(y_n - \hat{y})}{\sum_{n=1}^{N}(y_n - \hat{y})^2} \tag{3}$$

where $\hat{y} = \frac{1}{N}\sum_{n=1}^{N} y_n$ is the sample mean. The smallest positive value of T for which $\rho(T) \leq 0$ is often used as embedding lag. Data which exhibits a strong periodic component suggests a value for which the successive co-ordinates of the embedded data will be virtually uncorrelated whilst still being temporarily close.

The Lyapunov exponent is a measure of the rate of attraction to or repulsion from a fixed point in the phase space. One of the most prominent evidences of chaotic behavior of a chaotic system is the existence of positive Lyaponuv exponent. A positive Lyapunov exponent indicates divergence of trajectories in one direction, or alternaively, expansion of an initial volume in this direction, and a negative Lyapunov exponent indicates convergence of trajectories or contraction of volume along another direction. This average rate of divergence can be estimated by the method of Kantz (1994) which has been found to be efficient for short time series data and represented as.

$$\lambda_1 = \lim_{r \to \infty} \frac{1}{t} \ln\left(\frac{\Delta x(t)}{\Delta x(0)}\right) \tag{4}$$

The radio refractivity N is used instead of n and is expressed as;

$$N = 77.6\frac{P}{T} + 3.73 \times 10^5 \frac{e}{T^2} \tag{5}$$

where P = atmospheric pressure (hPa), e = water vapour pressure (hPa) and T = absolute temperature (K).

The water vapour pressure e is obtained from the relative humidity and saturated water vapour by the expression

$$e = H \times \frac{6.1121\exp(\frac{17.502t}{t+240.97})}{100} \quad (6)$$

where H = relative humidity (%), t = temperature in degree Celsius (^{o}C) and e_s = saturation vapour pressure (hPa).

The detailed description of the instrumentation set up and the experimental site where the data used for this work was obtained is available in Adediji and Ajewole (2008). Data were obtained at height 50m for the year 2007 - 2008.

RESULTS AND DISCUSSION

The radio refractivity estimated using equations (5) and (6) were plotted as a time series as depicted in Figure (1). It could be observed that phase space reconstruction will not be possible without obtaining the time delay and embedding dimension of the time series, hence the time delay was calculated using the method of mutual information given by equation (2). The result is presented in Figure (2). From the Figure, the first minimum was obtained at delay = 8. Hence, the time delay of 8 was used in this work. To obtain the embedding dimension, equation (3) was employed and the result is as shown in figure (3). A marked minimum was obtained at embedding dimension, therefore, the value of 9 was adopted as the embedding dimension throughout this work.

The phase space was reconstructed from the delayed time and the result is shown in Figure 4 which is a 3D representation. The time series of a non-chaotic system does not exhibit localized wave form but spread across the phase space. The concentration of the phase space around a local point is an evidence of nonlinearity in the refractivity index time series. To quantify and determine chaos in the time series, the Lyapunov exponent was computed using the algorithm of Kantz. From the Lyapunov exponent as presented in Figure 5, it can be deduced that the time series is chaotic as it has positive values.

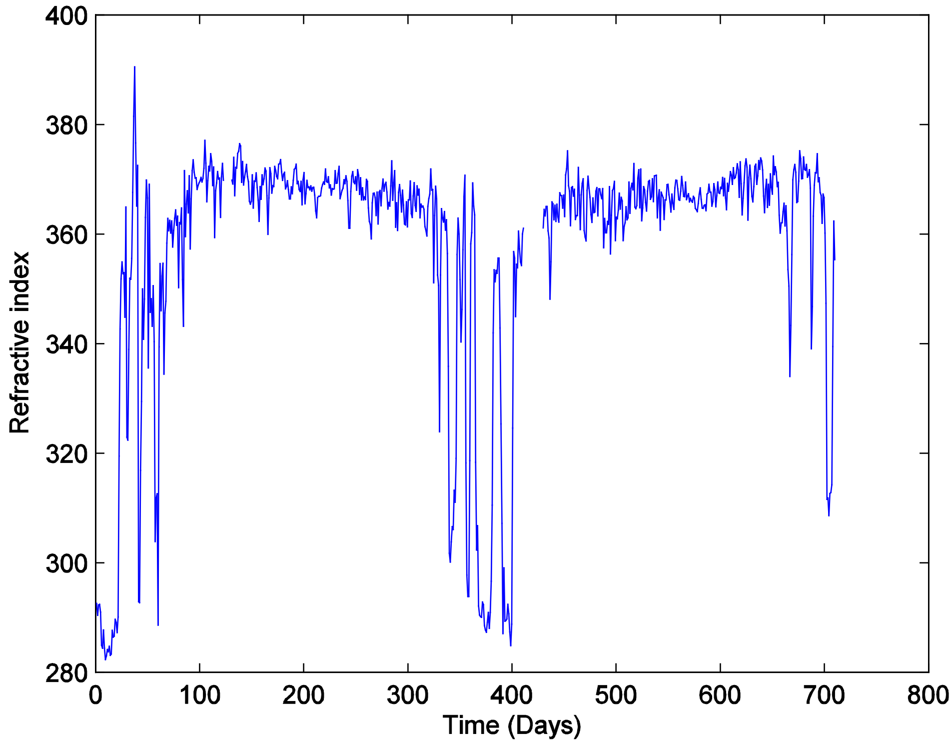

Figure 1: Time series of refractive index during the period 2007-2008 at height 50m

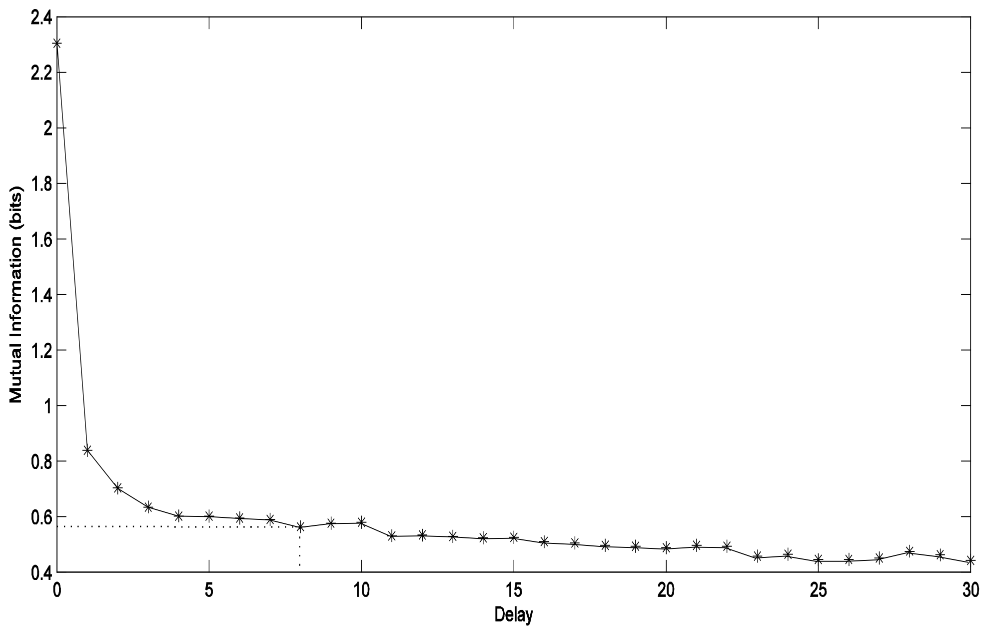

Figure 2: Time delay of refractive index using the method of mutual information.

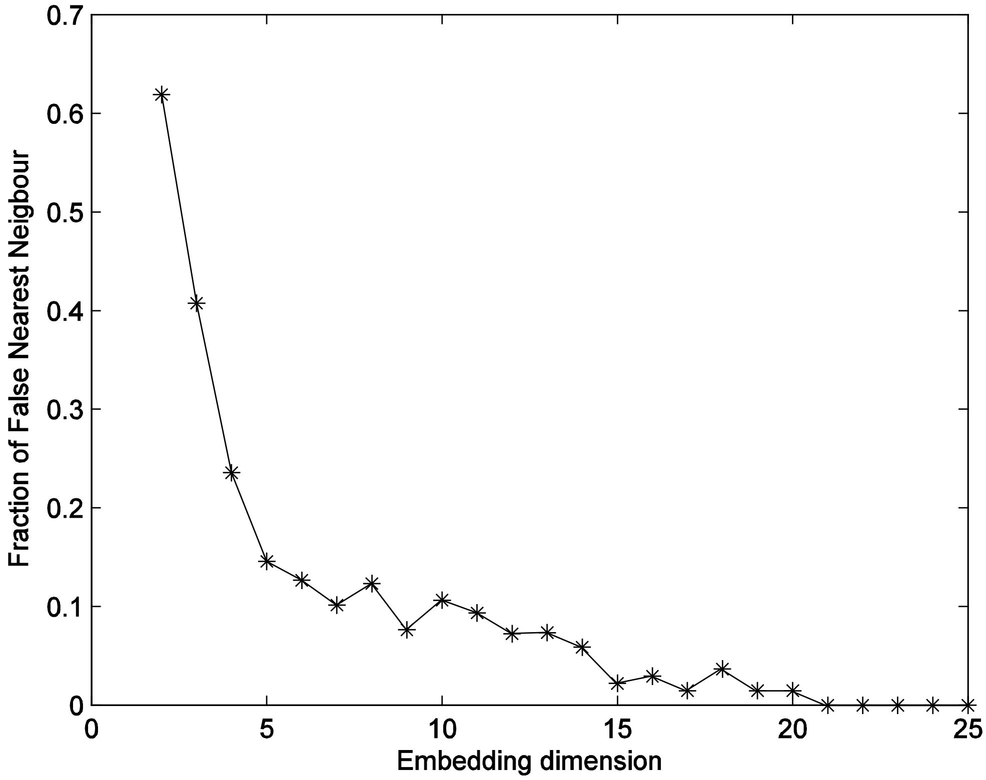

Figure 3: Embedding dimension using the method of False Nearest Neigbhour

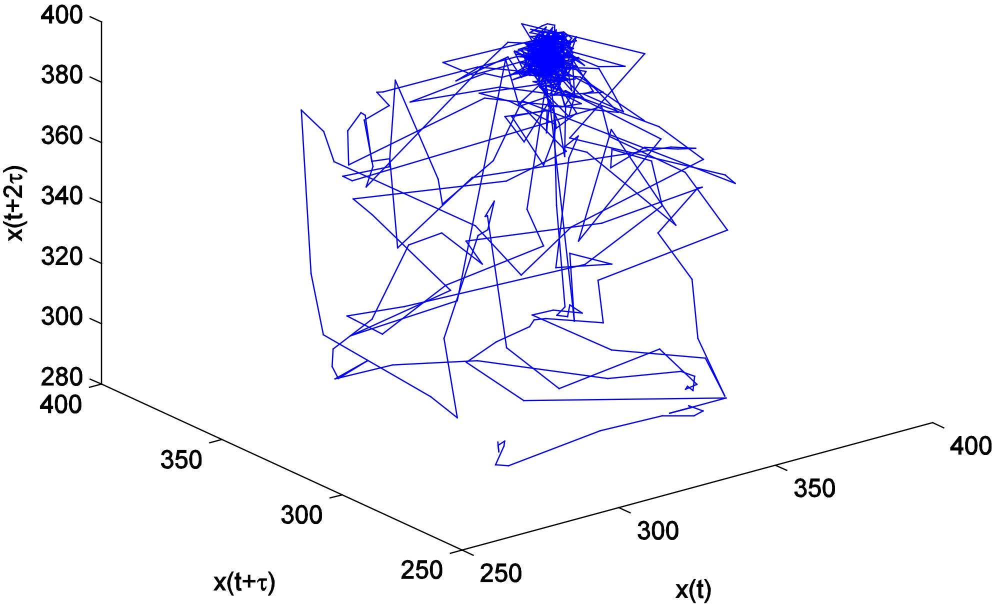

Figure 4: Phase space reconstruction from time series

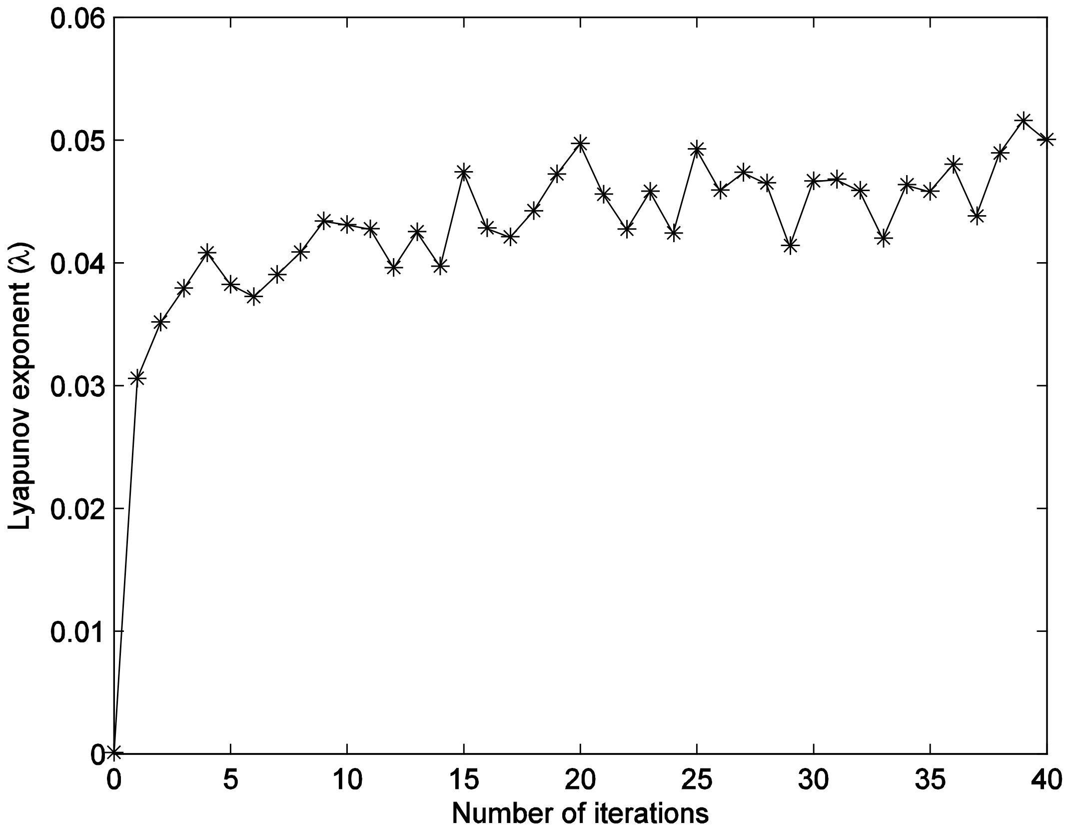

Figure 5: Lyapunov exponent

CONCLUSION

This research has investigated nonlinearity in the refractivity index time series over Akure, Southwestern Nigeria for a period of two years spanning 2007 - 2008. The time delay and embedding dimension were obtained from standard algorithms. The time delay was used to draw the phase space of the time series. Finally, the Lyapunov exponent was calculated. From the magnitude of the Lyapunov exponent, it can be inferred that there is low dimension chaos in the time series. This implies that long term prediction of the time series is impossible.

Hence, tropospheric radio propagation that are affected by variations in refractive index of the troposphere will have to use complex modelling technique to predict long term behaviour of radio refractivity. Hence, the need for better algorithm such as neural networks and nonlinear prediction methods for predicting the radio refractivity over a long period. Furthermore, other tools such as surrogate data analysis and recurrence plot can be used to further confirm the presence of chaos in the time series. The magnitude of the nonlinearity can be investigated by correlation integral and correlation dimension method.

References

Adediji, A.T. and Ajewole, M. O. (2008): Vertical profile of radio refractivity gradient in Akure South-West Nigeria. Progress in Electromagnetic Research C, Vol. 4. 157-168.

Adediji A.T., Ajewole M.O and Ojo J.S (2010) : Microwave Refractive Index Gradient and Effective Earth Radius Factor over Akure, Southwestern Nigeria. Conference Proceedings, Nigerian Union of Radio Science Annual Conference, Federal University of Technology, Akure, pp. 8 – 14.

Afullo, T. J, Motsoela, T and Molotsi, D.F (1999): Refractivity Gradient and k-factor in Botswana. Radio Africa. 107-110.

Bean, B.R. and Dutton, E.J. (1968): Radio Meteorology. Dover Publication Co. New York. 259- 273.

Derry G. N and Derry P. S (2010) Characterization of chaotic dynamics in the human menstrual cycle. Nonlinear Biomedical Physics 2010, 4:5

Falodun, S.E and Ajewole, M.O (2006): Radio refractive index in the lowest 100m layer of the troposphere in Akure, South Western Nigeria. Journal of Atmospheric and Solar-Terrestrial Physics, vol 68. 236-243

Fraser, A. M. and Swinney, H. L, (1986): Independent coordinates for strange attractors from mutual information. Physical Review A, 33(2), 1134-1140.

George, B. Renuka, G. , Satheesh Kumar, K., Anil Kumar,V, and Venugopal C. (2001): Nonlinear time series analysis of the fluctuations of the geomagnetic horizontal field. Annales Geophysicae 20: 175–183

Hsieh, D. A. (1993) : Implications of Nonlinear Dynamics for Financial Risk Management. *Journal of Financial and Quantitative Analysis,* 28(1): 41 – 64.

Kantz H (1994), *A robust method to estimate the maximal Lyapunov exponent of a time series*, Phys. Lett. A **185**, 77

Kortas Lukasz (2005): Search for chaotic dynamics manifestation in multiscale seismicity. Acta Geophysics Polonica.

Kyrtsou C and Labys W. (2006) Evidence for chaotic dependence between US inflation and commodity prices. Journal of Macroeconomics, 28(1), 256 – 266.

Li K and Gao Z. (2004) Nonlinear Dynamics Analysis of Traffic Time Series Modern Physics Letters B, Vol. 18, Nos. 26 & 27 1395–1402

Mundt, M. D., Maguire W.B. and Chase R.R.P. (1991) Chaos in sunspot cycle: Analysis and prediction. J. Geophysics Res. 96(A2) 1705 – 1716

Njah A. N and Ojo K.S (2010): Backstepping Control and Synchronization of parametrically and externally excited Φ^6 with application to secure communication. International Journal of Modern Physics B, 24(23) 4581 – 4593.

Ojo K. S and Ogunjo S. T. (2012): Synchronization of 4D Rabinovich Hyperchaotic Systems for secure communication. Journal of the Nigerian Association of Mathematical Physics, Vol 21, pp. 35 – 40.

Rapp, P. E., Zimmerman, I.D., Albano, A.M Degizman G. C. and Greenbaum N.N. (1985) Dynamics of spontaneous neural activity in the simian motor cortex: The dimension of chaotic neurons. Phys. Lett. A 110(6) 335-338.

Sarlis N. V. and Christopoulos S. R. G. (2012) Natural time analysis of the Centennial Earthquake Catalog Chaos 22, 023123

Sharifi, M.B. Georgakakos K.P. and Rodrigues-Iturbe I., (1990) Evidence of deterministic chaos in pulse of storm rainfall. J. Atmos. Sci 47(7) 888 - 893.

Shollykutty John and Kurian P. J. (2009) Chaos in the solar wind flow near Earth. *Pramana - J. Phys.,* Vol. 72(4)

Strogatz S.H. (1994), *Nonlinear Dynamics and Chaos* (Addison-Wesley, Reading 1994).

Yao H and Lui J (2010) Stability of a Time-Delay and Impulsive Financial model. International Journal of Nonlinear Science Vol (2010) N0 2, pp. 229 – 232.

Teh-Lu L and Shin_Hwa T (2000) Adaptive Synchronizatin of chaotic systems and its application to secure communication, Chaos, solitons and Fractals 11 (2000) 1387 – 1396.

Zhang, H., Holden, A. V Lab M. and Montonssis, M. (1992) Estimation of the persistence of strain from experimental recordings from cardiac tissue. Physica D 58(1-4) 489 – 496.