Electric machine is a general term for three different types of machines. These include: Electric motors, Electric generators and transformers. The action of all forms of Electromagnetic machines is based on two fundamental laws: Faraday’s law, which states that, whenever there is relative motion between the magnetic field and a coil situated in that field, then, there will be an e.m.f. induced in that coil, whose magnitude is equal to the time rate of change of flux linkage between the field and the coil.The second law is based on an observation by Maxwell who states that if a current-carrying conductor is placed in a magnetic field, a force will be exerted and relative motion will take place between the structure carrying the conductor and the structure carrying the field.

Oshin Ola Austin: A Christian and Power System Engineering lecturer in Elizade University is an effective and result oriented individual. He is an Electrical Engineer with proven integrity. He holds a Master Degree in Electrical Engineering and he's presently pursuing his PhD in Ladoke Akintola University of Technology, Nigeria.

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ELECTRICAL MACHINE THEORY

Fakorede (Ed.), Ola Austin

Oshin Ola Austin

Ebenezer Fakorede (Ed.)

Principles of Electrical Machines 1

Theory of Electrical Machines

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PRINCIPLE OF ELECTRICAL MACHINES 1

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ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ELIZADE UNIVERSITY, NIGERIA

2018

TITLE: PRINCIPLE OF ELECTRICAL MACHINES 1

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His contributions to knowledge and research work in Nigeria is worthy of good note.

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THESIS/DISSERTATION

1. Fault Evaluation and Improvement of Electric Power Distribution Network
   Federal University of Technology Akure, Nigeria 2014

LIST OF PUBLICATIONS BY OSHIN OLA AUSTIN

1. Incessant and Unreliable Power Supply in Nigeria: Causes, Effects and Solutions, LAP LAMBERT ACADEMIC PUBLISHING, Omni Scriptum GmbH & Co. KG, Deutschland Germany ISBN-13-978-3-659-74972-8 ** January, 2017
2. Basic Electrical Engineering, LAMBERT ACADEMIC PUBLISHING, Omni Scriptum GmbH & Co. KG, Deutschland Germany ISBN-978-3-330-71508-0 ** June,2017

KEY WORDS

Electric Motor, Dc Motors, Ac Motors Generator, Electrical Machines, Induction (Asynchronous) Motor and Synchronous Motor, Transformer, Rotating Magnetic Fields, Back E.M.F , E.M.F Generated in a Dc Generator, Performance and Method of Speed Control of Dc Machine, Characteristics of Dc Generators, Method of Speed Control of Dc Machine, Induction Motors, Principle of Operation of A Three-Phase Induction Motor, Rotor E.M.F, Losses in an Induction Motor, Torque Equation of an Electric Motor, Induction Motors, Asynchronous Motor, Armature Reaction in a D.C Machine, Cross Magnetism in Electrical Machines, Characteristics and uses of Series, Shunt and Cumulative Compound Motors

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CHAPTER ONE

INTRODUCTION

Electric machine is a general term for three different types of machines. These include: Electric motors, Electric generators and transformers. The action of all forms of Electromagnetic machines is based on two fundamental laws:

a. Faraday's law, which states that, whenever there is relative motion between the magnetic field and a coil situated in that field, then, there will be an e.m.f. induced in that coil, whose magnitude is equal to the time rate of change of flux linkage between the field and the coil.
b. The second law is based on an observation by Maxwell who states that if a current-carrying conductor is placed in a magnetic field, a force will be exerted and relative motion will take place between the structure carrying the conductor and the structure carrying the field.

ELECTRIC MOTOR

An electric motor converts electrical energy to mechanical energy. In a motor, magnetic field Ø will induce a magneto-motive force, F, on the current-carrying conductor placed within the field. The direction of the force is given by the Fleming's left-hand rule for motors which states that: 'if the left hand is held out with the first finger, second finger and thumb at right angle to each other. If forefinger or middle finger represents the direction of the line of force, second finger points or index finger in the direction of the induced current, then the thumb points in the direction of motion or applied force'. It commonly consists of two basic parts.

The stator or the yoke: This is an outside stationary stator having coils supplied with alternating current to produce a rotating magnetic field. The rotating magnetic field produce a magneto-motive force m.m.f which will be exerted on a conductor carrying current that has been placed between the structure carrying the conductor and the structure carrying the field. The rotor is attached to the output shaft. This rotor turns due to the torque exerted on it by the rotating field.

TYPES OF DC MOTOR

1. Permanent Magnet DC Motor
2. Separately Excited DC Motor
3. Self-Excited DC Motor are:
    a. Series wound DC
    b. Shunt wound DC
    c. Compound wound DC
    d. Short Shunt wound DC
    e. Long Shunt wound DC
    f. Differential Compound wound DC

TYPES AC MOTORS

1. Synchronous Motors :
2. Asynchronous Motor:

GENERATOR

An electric generator is a machine that converts mechanical energy into electrical energy. An electric generator works based on the principle that whenever conductor cuts the magnetic field, an E.M.F is induced on it and this will cause the current to flow through the conductor

TYPES OF DC GENERATORS

1. Permanent Magnet DC Generator
2. Separately Excited DC Generator
3. Self-Excited DC Generator which include the following:
    a. Series wound DC generator
    b. Shunt wound DC generator
    c. Compound wound DC generator

TYPES OF AC GENERATOR

1. Induction or asynchronous generator
2. Synchronous generator or alternator: In a synchronous generator, the rotor speed is synchronized with the frequency of the three phase or single phase supply. This speed is called synchronous speed.

TRANSFORMER

In a transformer, the energy received from the source at the primary winding or primary circuit, is first converted to magnetic energy and it is then reconverted to electrical energy in the secondary winding circuit, or third winding circuit etc.

There are three types of transformers

1. Step-up transformer
2. Step-down transformer
3. Isolation transformer

There are four types of transformers based on structure

1. core type
2. shell type
3. power type
4. instrument type

All these machines are electromechanical energy converters. This is because they convert energy from one form to another.

For example:

1. An electric motor converts electrical power to mechanical power
2. Electric generator converts mechanical power to electrical power.
3. In a transformer, the energy received from the source at the primary winding or primary circuit, is first converted to magnetic energy and it is then reconverted to electrical energy in the secondary winding circuit, or third winding circuit etc.

Electric Machine may have rotating parts and may not have any rotating parts. For instance, a transformer is an electric machine; but it has no moving or rotating parts like electric motors and generators.

When the moving parts in a machine are rotating, we call them rotating machines. When the moving parts in a machine is moving linearly, then we call them linear machines. Motors and generators have moving parts. The third category which is transformers, do not have any moving parts. They are still energy converters because they change the low voltage level of an alternating current to magnetic energy and it is then reconverted to electrical energy. Change of voltage may be from low voltage to a high voltage (step up transformer) or from high voltage to a low voltage (step down transformer).

Electric machines, in the form of generators, produce a large percentage of the electric power on Earth. Also, the electric motors consume about 60% of all electric power produced.

TWO MAIN PARTS OF AN ELECTRICAL MACHINE

The two main parts of an electrical machine can be described in two terms:

1. In Mechanical Terms
   Here, the rotor is the rotating part, and the stator is the stationary part of the electrical machine.
2. In Electrical Terms

   Here, the field is the magnetic field producing component and the armature is the mechanical power-producing component. The armature can be on either the rotor or the stator. The magnetic field can be provided in any of the two ways below:

   a. Electromagnets
   b. Permanent magnets mounted on either the rotor or the stator.

REVIEW OF ELECTRICAL ENERGY CONVERSION

THE GENERAL ELECTRICAL MACHINE

Electric machine is a general term for three different types of machines. They are as stated below:

ELECTRIC MOTORS

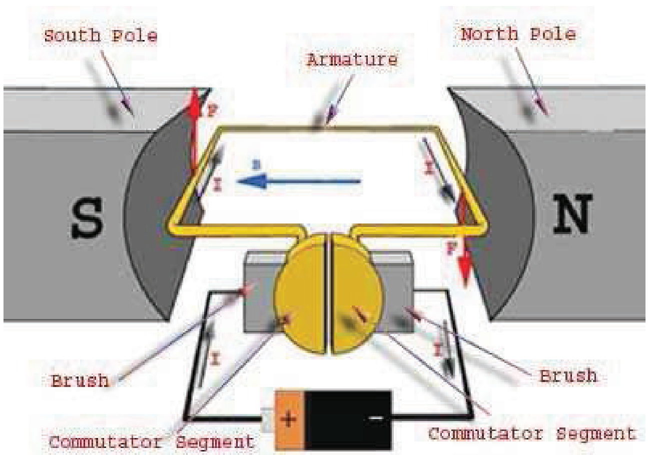


Figure 1.1: Operation of a DC motor

Reference: https://www.electrical4u.com/working-or-operating-principle-of-dc-motor/

The components of an electric motor are as shown in the figure below

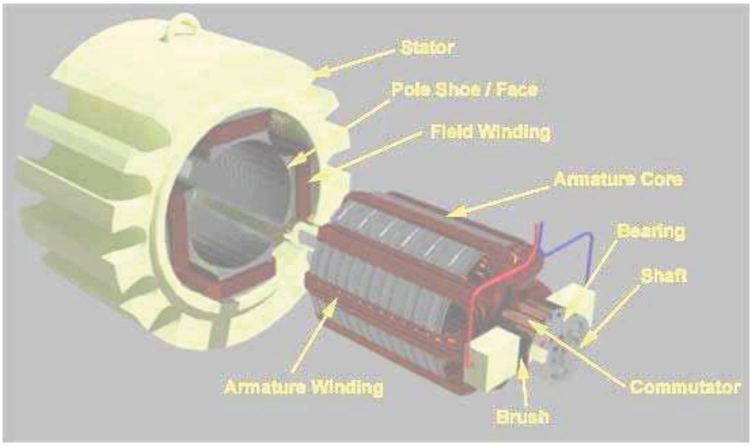


Figure 1.2: Components of DC motor

https://www.electrical4u.com/construction of dc motor yoke poles armature field winding commutator brushes of dc motor

COMPONENTS OF AN ELECTRIC MOTOR

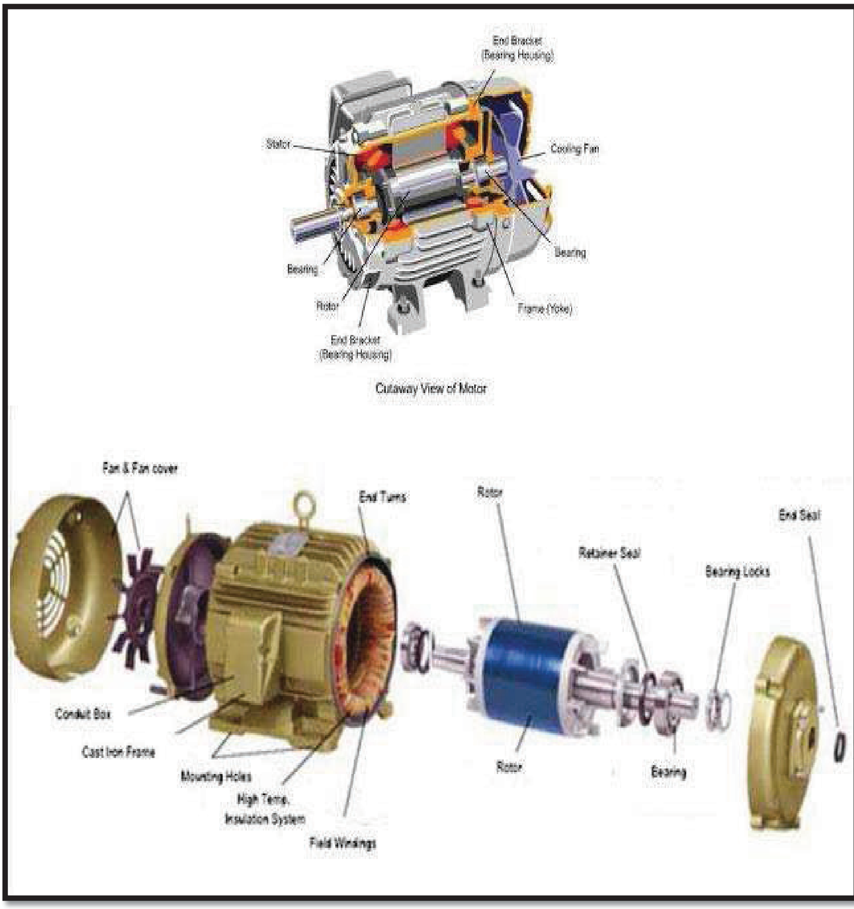


Figure 1.3: Components of DC motor

Reference: http://www.electrical-knowhow.com/2012/05/electrical-motors-basic-components.html

The constructional feature of an electric motor is shown below

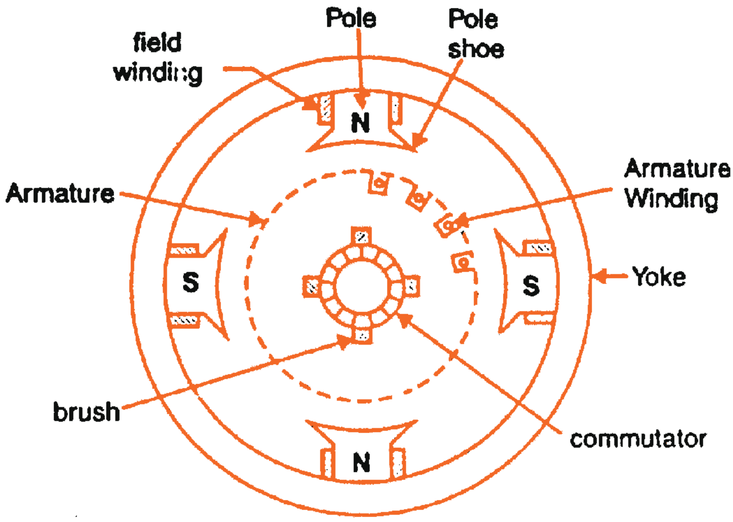


Figure 1.4 : showing the Constructional feature of a DC Machine

Reference: http://www.studyelectrical.com/2014/06/construction-dc-motor-dc-motor-construction.html

An electric motor has the following parts:

1. <u>Yoke</u>

The outer frame or yoke serves double purpose:

i. The yoke provides mechanical support for the poles. Also, it acts as a protecting cover for the whole machine
ii. It carries the magnetic flux produced by the poles.

Yokes are either made of cast iron or cast steel or rolled steel. When cheapness and not weight is the main consideration, for example in small generators, yokes are made of cast iron.

But for large machines, cast steel or rolled steel is usually used.

2. Pole Cores and Pole Shoes

The field magnets consist of pole cores and pole shoes. The pole shoes serve two purposes:

1. they spread out the flux in the air gap and also, being of larger cross-section, reduce the reluctance of the magnetic path
2. they support the exciting coils (or field coils)

3. Field system

The field system produces uniform magnetic field within which the armature rotates.

Field coils are mounted on the poles and carry the dc exciting current. The field coils are connected in such a way that adjacent poles have opposite polarity.

The m.m.f. developed by the field coils produces a magnetic flux that passes through the pole pieces, the air gap, the armature and the frame.

Practical d.c. machines have air gaps ranging from 0.5 mm to 1.5 mm.

Electric Motor

An electric motor converts electrical energy into mechanical energy.

Electric motors operate through interacting magnetic fields and current-carrying conductors. When the current carrying conductor is situated within the magnetic field, a rotational magnetic force will be produced in the motor.

Motors and generators have many similarities and many types of electric motors can be run as generators, and vice versa.

Figure 1.5 : ELECTRIC MOTOR

Reference: Stator and Rotor by Zureks. JPG

ELECTRIC MOTOR

Electric motors can be used in the following electrical systems:

1. power tools,
2. industrial fans,
3. blowers and pumps,
4. machine tools,
5. household appliances,
6. Disk drives.

They may be powered by direct current or by alternating current. This will then lead to two types of motor:

1. DC motors
2. AC motors.

DC MOTOR

The brushed DC electric motor generates torque directly from DC power supplied to the motor by using internal commutation, stationary permanent magnets, and rotating electrical magnets. Brushes and springs carry the electric current from the commutator to the spinning wire windings of the rotor inside the motor. Brushless DC motors use a rotating permanent magnet in the rotor, and stationary electrical magnets on the motor housing. A motor controller converts DC to AC. This design is simpler than that of brushed motors because it eliminates the complication of transferring power from outside the motor to the spinning rotor. An example of a brushless, synchronous DC motor is a stepper motor which can divide a full rotation into a large number of steps.

TYPES OF DC MOTOR

1. Permanent Magnet DC Motor
2. Separately Excited DC Motor
3. Self-Excited DC Motor are:
    g. Series wound DC
    h. Shunt wound DC
    i. Compound wound DC
    j. Short Shunt wound DC
    k. Long Shunt wound DC
    l. Differential Compound wound DC

AC MOTORS

TYPES AC MOTORS

The following motors are the types of motors and their examples:

3. Synchronous Motors :
    a. Plain Synchronous Motors
    b. Super Synchronous Motors
4. Asynchronous Motor:
    a. Induction Motor:
        i. Squirrel cage induction motor
        ii. Slip ring external resistance or wound motor
    b. Commutator Motors
        i. Series Commutator Motors
        ii. Compensated Commutator Motors
        iii. Shunt Commutator Motors
        iv. Repulsion Commutator Motors
        v. Repulsion Start Commutator Motors

An AC motor converts alternating current into mechanical energy. It commonly consists of two basic parts.

1. The stator or the yoke: This is an outside stationary stator having coils supplied with alternating current to produce a rotating magnetic field. The rotating magnetic field produce a magneto-motive force m.m.f which will be exerted on a conductor carrying current that has been placed between the structure carrying the conductor and the structure carrying the field. The rotor is attached to the output shaft. This rotor turns due to the torque exerted on it by the rotating field.

The torque T developed in this motor is given by:

$$T = \frac{p\emptyset Z I_a}{\pi C}$$

Where,

1. $p = number\ of\ pairs\ of\ pole$
2. $\emptyset = flux\ per\ pole$
3. Z = number of conductors
4. I_a =current from the source connected to the stator winding or the armature winding. *And*

$$\pi = 3.142$$

5. c = number of parallel paths

For wave winding c=2, but for lap winding c= 2 p

INDUCTION (ASYNCHRONOUS) MOTOR AND SYNCHRONOUS MOTOR

1. Induction (asynchronous) motor: In induction (asynchronous) motor, the rotor magnetic field is created by an induced current. The rotor must turn slightly slower (or faster) than the stator magnetic field to provide the induced current.
   There are three types of induction motor rotors
   a. Squirrel-cage rotor
   b. Wound rotor and
   c. Solid core rotor.
2. Synchronous motor does not rely on induction and so can rotate exactly at the supply frequency or sub-multiple. The magnetic field of the rotor is either generated by direct current (this is called external exciter) delivered through slip rings or by a permanent magnet.

CHAPTER TWO

ELECTRIC GENERATOR

An electric generator is a device that converts mechanical energy to electrical energy.

The source of mechanical energy, the prime mover, may be from any of the following sources:

1. Internal combustion engine
2. A reciprocating or turbine steam engine
3. Water falling through a turbine or waterwheel
4. A wind turbine,
5. A hand crank
6. Compressed air or any other source of mechanical energy

GENERATORS ARE CLASSIFIED INTO TWO TYPES

1. AC generators and
2. DC generators.

AC GENERATOR

An AC generator converts mechanical energy into alternating current electricity. Many AC generators have the rotating field winding on the rotor and the armature winding on which electric power is produced on the stator. This is because, power transferred into the field circuit is much less than power transferred into the armature circuit.

AC generators are classified into the following types:

1. **Induction Generator**: Induction Generators are singly excited. An alternating current source supply current to only the stator of the machine. Then, stator magnetic flux will be generated and the stator magnetic flux induces currents in the rotor and rotor magnetic flux will be set up. There is no physical connection between the stator windings and the field windings. According to Lenz law, the rotor flux will act in opposite direction to the stator flux that produces it. The rotor then rotates. At this stage, the machine acts as a motor.

Later, the prime mover then drives the rotor above the synchronous speed. What will happen is that the opposing rotor flux cut the stator coils again. At the end, active current will be generated on the stator coil. The generated power will then be delivered to the mains from which it was taking power when operating as a 3-phase induction motor.

> ***Take note that an induction generator draws reactive power from the connected system (current source) and so cannot be an isolated source of power.***

2. **Synchronous generator or alternator**: In a synchronous generator or alternator, the current for the magnetic field is provided by a separate DC current source. In a synchronous motor, the rotor speed is synchronized with the frequency of the three phase or single phase supply. This speed is called synchronous speed.

DC GENERATOR

An electric generator is a machine that converts mechanical energy into electrical energy. An electric generator works based on the principle that whenever conductor cuts the magnetic field, an e.m.f is induced which will cause the current to flow if conductor circuit is closed.

TYPES OF DC GENERATORS

4. Permanent Magnet DC Generator
5. Separately Excited DC Generator
6. Self-Excited DC Generator which include the following:
    d. Series wound DC generator
    e. Shunt wound DC generator
    f. Compound wound DC generator

TRANSFORMER

A transformer is a static device that converts alternating current from one voltage level to another level without changing the frequency.

A transformer transfers electrical energy from one circuit to another through the transformer windings (inductively coupled conductors, i.e. the transformer's coils. A varying electric current in the first or *primary* winding creates a varying magnetic flux in the transformer's core. Hence, a varying magnetic field through the *secondary* winding will be created. This varying magnetic field induces a varying electromotive force (EMF) or voltage in the secondary winding.

This effect is called mutual induction.

There are three types of transformers

4. Step-up transformer
5. Step-down transformer
6. Isolation transformer

There are four types of transformers based on structure

5. core type
6. shell type
7. power type
8. instrument type

ROTATING MAGNETIC FIELDS

The action of all forms of Electromagnetic machines is based on two fundamental laws:

c. Faraday's law, which states that, when there is relative motion between the magnetic field and a coil situated in that field, there will be an e.m.f. induced in that coil, whose magnitude is equal to the time rate of change of flux linkage between the field and the coil.

d. The second law is based on an observation by Maxwell who states that if a current-carrying conductor is placed in a magnetic field, a force will be exerted and relative motion will take place between the structure carrying the conductor and the structure carrying the field.

There will be two electrical members of an electrical machine:

1 The field winding and
2. The armature winding in which work is done as shown below:

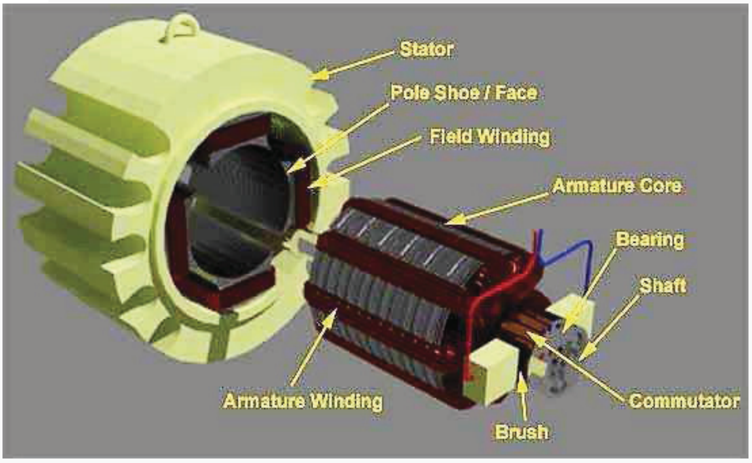


Figure 2.1 : Physical structure and configuration of DC Machines

Reference: https://www.electrical4u.com/construction-of-dc-motor-yoke-poles-armature-field-winding-commutator-brushes-of-dc-motor/

TWO-POLE MACHINE

The figure below represents a two-pole machine in which the stator poles are constructed in such a way as to project closer to the rotor than to the stator structure. This type of construction is rather common, and poles constructed in this fashion are called salient poles.
At the same time, the rotor could also be constructed to have salient poles.

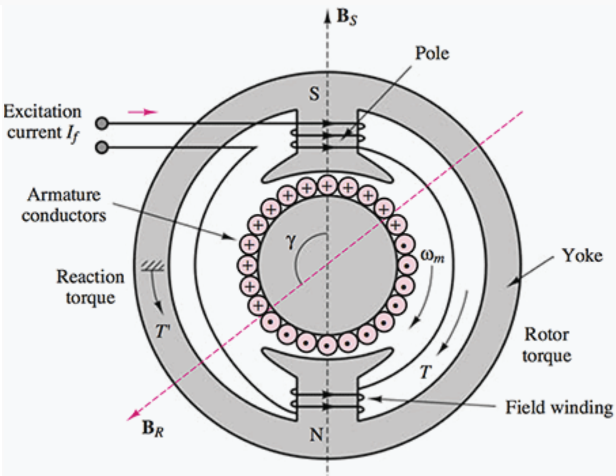


Figure2.2 : Cross section of DC machine
Reference: Physical Structure and configuration of DC Machines by Edvard
http://electrical-engineering-portal.com/physical-structure-configuration-of-dc-machines

The armature winding and the commutator are on the rotor. The brushes are pressed on the commutator surface rotating with the rotor.
The brushes take power from a DC source and feed current I_a to the commutator and the armature winding on the rotor.
The field winding is located on the stator. It is connected to an external DC supply that supplies current I_f to the field winding. Hence, the single phase or three phase set of current, flowing in the armature windings each has equal magnitude. The currentI_f flowing through the field winding creates a magnetic flux of density B.

PRODUCTION OF MAGNETIC FIELD

Ampere's law which is the basic law which governs the production of magnetic field provided the magnetic field intensity that will be produced by current I_f as shown below:

$$Hl_c = NI_f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where,

l_c = mean path length of the coil

H = Magnetic field intensity produced by the current I_f

I_f = field current, from the external DC supply

N = Number of turns of wire on the stator

The magnetic field intensity is related with the resulting magnetic flux density B as shown below:

$$B = \mu H \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where

H = Magnetic field intensity in ampere-turns per meter

B = Magnetic field density in Weber/meter-square known as Tesla

μ = magnetic permeability of the material. Magnetic permeability is the relative ease of establishing a magnetic field in a given material

The total magnetic flux in the core is Ø.

And

$$Ø = BA \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Where

Ø = magnetic flux

B = Magnetic field density and

A = cross sectional area of the core

Comparing the three equations above, we have

$$Ø = BA$$

$$Ø = \mu H A$$

$$Ø = \frac{\mu N I_f A}{l_c}$$

$$B = \frac{\mu N I_f}{l_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The armature winding will be fed from a DC supply. For a DC motor, Maxwell law states that if a current-carrying conductor is placed in a magnetic field, a force will be exerted and relative motion will take place between the structure carrying the conductor and the structure carrying the field.

IN A MOTOR

The magnetic field that is Ø will induce a magneto-motive force, F, on the current-carrying conductor placed within the field. The direction of the force is given by the Fleming's left-hand rule for motors which states that: 'if the left hand is held out with the first finger, second finger and thumb at right angle to each other. If forefinger or middle finger represents the direction of the line of force, second finger points or index finger in the direction of the induced current, then the thumb points in the direction of motion or applied force as shown in the below

$$\mathbf{F = Bi}\ \boldsymbol{l_c}\mathbf{sin\ \theta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

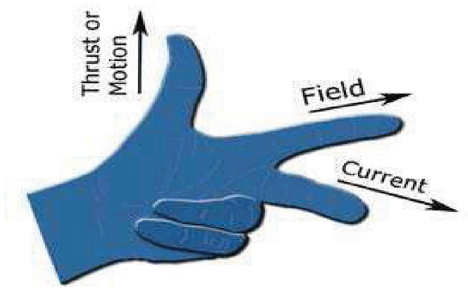


Figure2.3 Fleming's left hand rule

Reference: Douglas ,Morrison Dough, https://en.wikipedia.org/wiki/Fleming%27s_left-hand_rule_for_motors#/media/File:LeftHandOutline.png

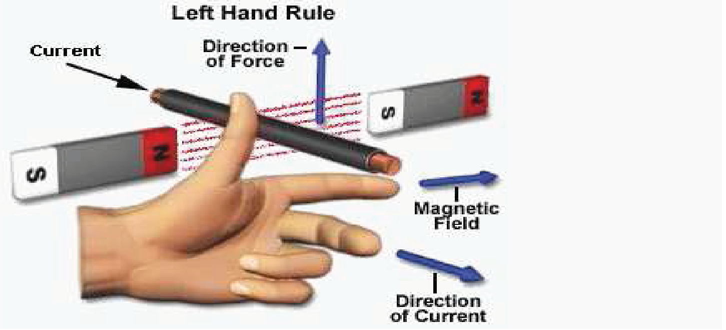


Figure2.4 : Fleming's left-hand rule (for motors)

Reference: Douglas ,Morrison Dough, https://en.wikipedia.org/wiki/Fleming%27s_left-hand_rule_for_motors#/media/File:LeftHandOutline.png

The force produces a torque T which can be obtained as follows:

For a DC motor, if the applied voltage is V. When the DC motor rotates, an e.m.f will be induced in the armature windings. This induced e.m.f is E. From Lenz law, the induced e.m.f , E will oppose the motion of the supply voltage that produces it.

This e.m.f, E is known as back e.m.f . It is less than the applied voltage as shown in the equation below:

$$E = V - I_a R_a \qquad \text{.................................. (6)}$$

Hence

$$V = E + I_a R_a \qquad \text{.................................. (7)}$$

If we multiply each term in the equation by I_a , then we will obtain

$$VI_a = EI_a + I_a R_a I_a$$

$$VI_a = EI_a + I_a.^2 R_a \qquad \text{..................................(8)}$$

Generally in an electric motor,

Electrical power = Mechanical Power + Losses

$$VI_a = \text{ Electrical power}$$

EI_a = Mechanical Power ……………………………..(9)

$I_a.^2\, R_a$= Lossesdue to the armature resistance R_a

Also,

The mechanical power is = torque (T) in Newton-metres x mechanical speed of the motor (w)

Mechanical power is = T x w ……………………….. (10)

Comparing 9 and 10

EI_a= T x w since w = $2\pi f$ or w = $2\pi n$

n = armature speed in rev/sec

f =Frequency of the motor in rev/sec

Take note that if the loss in the shunt circuit and sum of the iron, friction and windage losses are neglected

Then, armature speed in rev/sec= the frequency of the rotor in rev/sec

Then,

EI_a = T x $2\pi n$

$$T = \frac{EI_a}{2\pi n}$$

THE BACK E.M.F

The e.m.f generated by the armature is equal to the e.m.f generated by one of the parallel paths.

During one revolution, each conductor will pass through P poles.

The total magnetic flux in the core per pole is Ø.

1 pole = Ø weber

Therefore, each conductor will cut a total flux of P x Ø weber per revolution

1 pole = Ø weber

P poles =?

Then, total number of flux inn one revolution =P Ø weber

In one revolution, total flux is P Ø weber

The armature makes n revolutions in one second. So, n is in revolution per second

If one revolution = P Ø weber

Then, n revolutions = Y

Total flux in one second = Y = P Ø n Weber

It is important for you to know that 1 volt = 1weber per second

Hence, the e.m.f generated per conductor can now be calculated as follows:

E = P Ø n volts P = number of poles

Also, E = 2*p*Ø n volts *p* = number of pairs of poles, P = 2*p*

There are 2 brushes pressed against the commutator in a DC machine

The number of conductors in series in each path = $\frac{Z}{c}$

Where,

Z = number of conductor in series

c = number of parallel paths

Ø = useful flux per pole in Weber

c = 2, for a wave winding

c = 2p or P, in a lap winding

Hence, the total e.m.f generated = average e.m.f generated /conductor x number of conductor in series per phase

$$E = P\,Ø\,n \times \frac{Z}{c} \quad \text{volts}$$

$$E = \frac{PØnZ}{C} \quad \text{or} \quad E = \frac{2pØnZ}{C}$$

Since

$$T = \frac{EI_a}{2\pi n}$$

$$T = \frac{\frac{PØnZ}{C} I_a}{2\pi n} \quad \text{or} \quad T = \frac{\frac{2pØnZ}{C} I_a}{2\pi n}$$

$$T = \frac{PØnZ}{2\pi nC} I_a \quad \text{or} \quad T = \frac{2pØnZ}{2\pi nC} I_a$$

$$T = \frac{PØZ}{2\pi C} I_a \quad \text{or} \quad T = \frac{pØZ}{\pi C} I_a$$

The armature makes n revolutions in one second. So, n is in revolution per second

Let the number of pole P, number of pairs of pole p, number of conductors in series Z, π and the number of parallel paths gives a constant value k. Then,

$$T = kØI_a, \; k = \frac{PZ}{2\pi C}$$

or $T = kØI_a$, $k = \frac{pZ}{\pi C}$

Where

1. $p = number\ of\ pairs\ of\ pole = \frac{P}{2}$, P = number of poles
2. Ø = flux per pole in Weber
3. Z = number of conductors
3. I_a =current from the source connected to the stator Winding or the armature winding
4. $\pi = 3.142$
5. c number of parallel = paths

For wave winding c=2, but for lap winding c= 2 p

FOR GENERATORS:

Fleming's right-hand rule reveals that: if you hold out the right hand with the first finger, second finger and thumb at right angle to each other as shown below; with the forefinger represents the direction of the line of force, the thumb points in the direction of motion or applied force, then second finger will point in the direction of the induced current as shown below.

Fleming Right Hand Rule

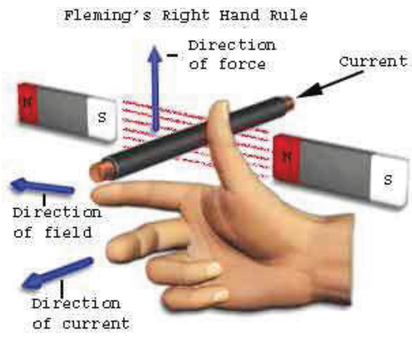


Figure2.5 : Fleming Right Hand Rule
Reference: Douglas ,Morrison Dough, https://en.wikipedia.org/wiki/Fleming%27s_left-hand_rule_for_motors#/media/File:LeftHandOutline.png

They were originated by John Ambrose Fleming, in the late 19th century, as a simple way of working out the direction of motion in an electric motor, or the direction of electric current in an electric generator.

Note that Fleming's right-hand rule is for generators. While, Fleming's left-hand rule is for motors.

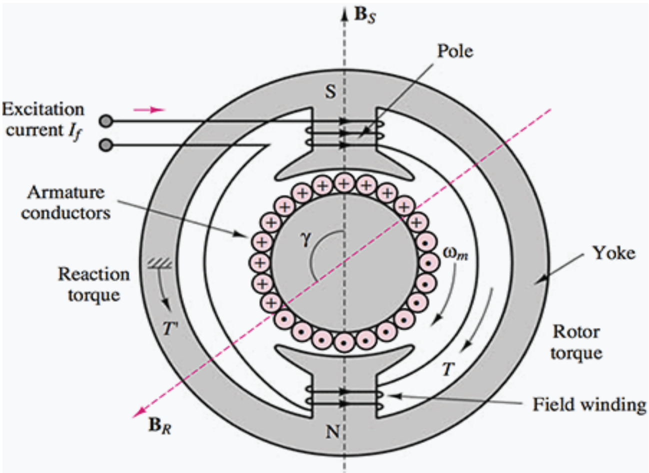


Figure2.6 : **Cross section of DC machine**
Reference: Physical Structure and configuration of DC Machines by Edvard
http://electrical-engineering-portal.com/physical-structure-configuration-of-dc-machines

EMF GENERATED IN A DC GENERATOR

When a load is connected across the armature of terminals, a load current, I_a will flow.

The EMF E generated will fall to V (the terminal voltage) due to the armature resistance, R_a.

$$V = E - I_a R_a$$

$$E = V + I_a R_a$$

The e.m.f generated by the armature is equal to the e.m.f generated by one of the parallel paths.
During one revolution, each conductor will pass through P poles.
The total magnetic flux in the core per pole is Ø.

1 pole = Ø weber

Therefore, each conductor will cut a total flux of P xØ weber per revolution

1 pole = Ø weber
P poles =?

Then, total number of flux in one revolution =P Ø weber

In one revolution, total flux is P Ø weber

The armature makes n revolutions in one second.

if one revolution = P Ø weber

Then, n revolutions = **x** *weber*

Total flux in one second= **x** = P Ø n weber

It is important for you to know that 1 volt = 1weber per second

Hence, the generated e.m.f per conductor can now be calculated as follows:

E = P Ø n volts P = number of poles

Also, E = 2*p*Ø n volts *p* = number of pairs of poles, P = 2*p*

There are 2 brushes pressed against the commutator in a DC machine

The number of conductors in series in each path = $\frac{Z}{c}$

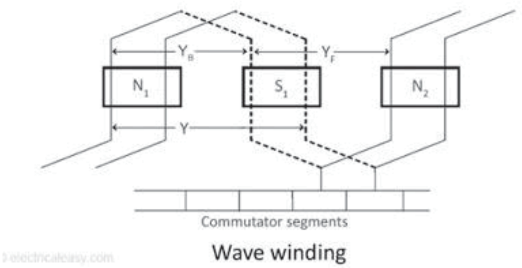
Where,

Z = number of conductor in series

c = number of parallel paths

c = 2, for a wave winding

c = 2p or P, in a lap winding



c = 2, for a wave winding

Figure2.7 : **wave winding**

Armature Winding of a DC Machine, By Kiran Daware, Kiran, Daware and www.electricaleasy.com

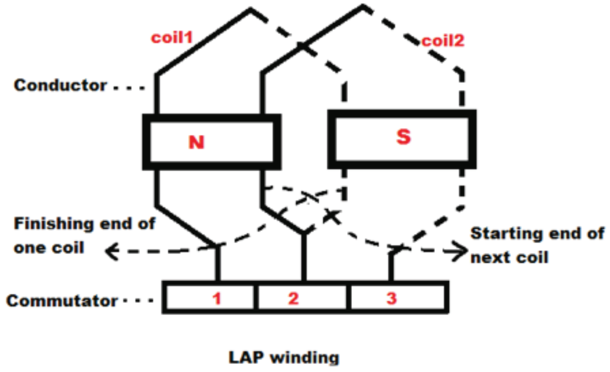
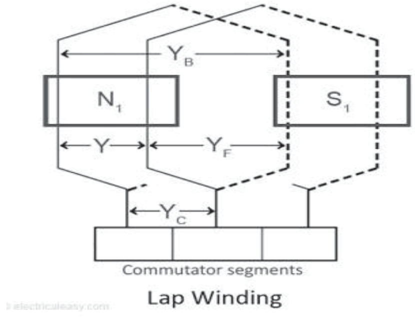


Figure 2.8 : Lap winding

c = 2*p* or P, in a lap winding

p = number of pairs of pole P = Number of poles, $p = \frac{P}{2}$

That means if P =8, $p = \frac{8}{2} = 4$



c = 2p or P, in a lap winding

Figure 2.9: Armature Winding of a DC Machine, Kiran Daware, www.electricaleasy.com **and** https://www.quora.com

Hence, the total e.m.f generated = average e.m.f generated /conductor x number of conductor in series in each path

$$E = P\,Ø\,n \times \frac{Z}{C} \quad \text{volts}$$

$$E = \frac{PØnZ}{C} \quad \text{or} \quad E = \frac{2pØnZ}{C}$$

Let the number of pole P, number of pairs of pole p, number of conductors in series Z and the number of parallel paths gives a constant value k. Then,

$$E = kØn \;, \qquad k = \frac{PZ}{C}$$

or

$$E = kØn \;, \qquad k = \frac{2pZ}{C}$$

The armature makes *n* revolutions in one second. So, *n* is in revolution per second. From elementary that means n is the same thing as frequency f in rev/sec or hertz

Hence,

If the mechanical speed of the generator is ω in radian/sec

Then $\omega = 2\pi n$

$$n = \frac{\omega}{2\pi}$$

Since,

$$E = \frac{PØnZ}{C} \quad \text{or} \quad E = \frac{2pØnZ}{C} \text{ n is in rev/sec}$$

Then,

$$E = \frac{P\text{Ø}\omega Z}{2\pi C} \quad \text{or} \quad E = \frac{2p\text{Ø}\omega Z}{2\pi C}$$ **ω is in radian/sec**

Here, armature constant K_a will be

$K_a = \frac{PZ}{2\pi C}$ P = number of poles

$K_a = \frac{pZ}{\pi C}$ p = number of poles

And P = 2 p

Direct Current Motor (Rotor, Stator)

A representative DC machine is shown in the figure above. The magnetic poles clearly identified, for both the stator and the rotor.

Note the salient pole construction of the stator and the slotted rotor. As stated earlier, the torque developed by the machine is a consequence of the magnetic forces between stator and rotor poles.

In a synchronous machine, a magnetic flux density B_R is produced by the rotor, and a magnetic flux density B_s is produced by the stator as shown below:

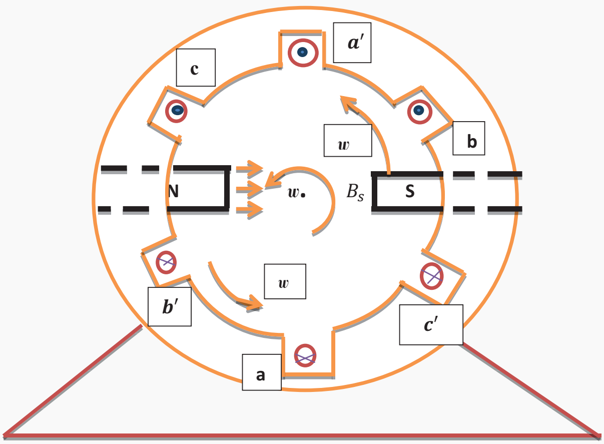


Figure 2.10 : The production of rotating magnetic field in a stator represented as moving north and south stator poles

A four poles electrical machine means a 2 north poles and 2 south poles machine.

A coil has 2 ends: a starting end and finishing end. A winding may have one or more turns. A winding will always have 2 ends even if it has only one turn.

So, 6 windings will have 12 ends altogether: 6 starting and 6 finishing.

A six winding electrical machine will produce the same magnetic field as a 4 poles electrical machine. Therefore, 4 pole electrical; machine is the same as six winding electrical machine. ***In practice, when a three phase set of current is applied to the stator windings of 4 poles or six windings AC or DC machine, North and South poles will be created in the machine, the pole will move around(rotate around) half the stator surface or half revolution i.e 180^o in one electrical surface.***

Since the mechanical motion in 4 poles electrical machine is 180^o for every electrical cycle of 360^o.

Then in 4poles electrical machine,

The electrical angle θ_e of 360^o is = 2 x mechanical angle θ_m of 360^o

or electrical angle θ_e is $= \frac{4}{2}$ x mechanical angle θ_m

or electrical angle θ_e is $= \frac{P}{2}$ x mechanical angle θ_m

Finally,

$$\theta_e = 2\,\theta_m \text{ for 4 poles}$$

Since frequency means the number of revolutions completed in one second, the electrical frequency is also double the mechanical frequency for 4 poles

$$f_e = 2\,f_m \text{ for 4 poles}$$

Since angular velocity $w = 2\pi f$

the electrical angular velocity is also double the mechanical angular velocity for 4 poles

$$w_e = 2\,w_m \text{ for 4 poles}$$

TAKE NOTE THAT:

1. If there are P numbers of magnetic poles on the stator of an electromagnetic machine, then there will be $\frac{P}{2}$ repetition of the windings.

2. $\theta_e = \frac{P}{2} \times \theta_m$

3. $f_e = \frac{P}{2} f_m$

4. $w_e = \frac{P}{2} w_m$

5. Mechanical frequency = f_m in revolution per second

   Mechanical speed in revolution per minute = N_m

   Then,

$$f_m = \frac{N_m}{60}$$

6. Comparing number 3 and 5 above

$$f_e = \frac{P}{2} f_m \; , \qquad f_m = \frac{N_m}{60}$$

$$f_e = \frac{P}{2} \times \frac{N_m}{60}$$

$$f_e = \frac{P\,N_m}{120}$$

$$N_m = \frac{120 f_e}{P}$$ P= number of poles

7. Also, $N_m = \frac{60 f_e}{p}$

p = number of pairs of pole i.e $p = \frac{P}{2}$

8. Let the mechanical speed of the machine in revolution per second

   be= n_m

   Then,

9. $n_m = \frac{N_m}{60}$

Since $N_m = \frac{60f_e}{p}$

Then, $n_m = \frac{60f_e}{px60}$

Hence,

10. $n_m = \frac{f_e}{p}$

Example 1

An 8 pole, wave-connected armature has 700 conductors and is driven at 720 rev/min. determine the generated E.M.F when the flux per pole is 22mWb

Since,

$E = \frac{PØnZ}{C}$ or $E = \frac{2pØnZ}{C}$

P = number of pole = 8

$p = number\ of\ pairs\ of\ pole = \frac{P}{2} = \frac{8}{2} = 4$

Ø = flux per pole = 22mWb= $22x10^{-3}$ Wb

n = speed of the machine in rev/sec

Since the speed N=720 rev/min

Then, $n = \frac{N}{60}$

$n = \frac{720}{60}$

n = 12 rev/sec

Z = number of conductors = 700

c = number of parallel path

c = number of parallel paths

c = 2, for a wave winding

c = 2*p* or c= P, in a lap winding, *p = number of pairs of poles*

P = Number of poles

Finally,

$$E = \frac{P\O nZ}{C} \quad \text{or} \quad E = \frac{2p\O nZ}{c}$$

$$E = \frac{8x\ 22\ x\ \frac{720}{60} x\ 700}{2} \quad \text{or} \quad E = \frac{2x4x22x10^{-3}x\ \frac{720}{60} x700}{2}$$

$$E = \frac{1478400x10^{-3}}{2} \quad \text{or} \quad E = \frac{1478400x10^{-3}}{2}$$

$$E = \frac{1478.40}{2} \quad \text{or} \quad E = \frac{1478.40}{2}$$

E = 739.2Volts or E =739.2 Volts

Example 2

An 8-pole generator has a lap-wound armature with 50 slots with 20 conductors per slot. The useful flux per pole is 24mWb. Determine the speed at which the machine must be driven to generate an e.m.f. of 240 V

Solution

The total e.m.f generated in a generator = average e.m.f generated /conductor x number of conductor in series in each path

$$\text{i.e } E = P\,Ø\,n \;\; x\frac{Z}{c} \qquad \text{volts}$$

$$E = \frac{PØnZ}{c} \;\; \text{or} \;\; E = \frac{2pØnZ}{c}$$

e.m.f E = 240V, (for a lap winding), speed in rev/sec = n,

Flux per pole Ø= 24mWb= 24 x 10^{-3},

Z = number of conductors = number of conductors/slot x number of slot

Z = number of conductors = 20 x 50

Z = number of conductors = 1000

Number of parallel path = c =2p $\quad p = number\ of\ pairs\ of\ pole$

Number of parallel path = c =P P= number of poles

Since,

$$E = \frac{PØnZ}{C} \;\; \text{or} \;\; E = \frac{2pØnZ}{C} = \frac{2pØnZ}{2p}$$

$$240 = \frac{8x24\text{ x }10^{-3}x\text{ n x }1000}{8} \;\; \text{or} \;\; 240 = \frac{8x24\text{ x }10^{-3}x\text{ n x }1000}{2x4}$$

$$240 = \frac{8x24\text{ x }10^{-3}x\text{ n x }1000}{8} \;\; \text{or} \;\; 240 = \frac{8x24\text{ x }10^{-3}x\text{ n x }1000}{2x4}$$

$$240 = \frac{192\text{ n}}{8} \quad \text{or} \quad 240 = \frac{192\text{n}}{8}$$

n=10 rev/sec or 10 x 60 = 600 rev/sec

Example 3

A 4-pole, lap-wound armature has 1600 conductors and a flux per pole of 0.02Wb. Determine

2. The e.m.f. generated when the machine is running at 600rev/min.
3. the e.m.f generated if the armature is wave wound

Solution

1. for a lap winding,

Number of parallel path = c =$2p$ $p = number\ of\ pairs\ of\ pole$

orNumber of parallel path = c =P $\text{P} = number\ of\ poles$

Since,

$$\text{E} = \frac{\text{PØnZ}}{C} \quad \text{or} \quad \text{E} = \frac{2p\text{ØnZ}}{C}$$

$$\text{E} = \frac{\text{PØnZ}}{\text{P}} \quad \text{or} \quad \text{E} = \frac{2p\text{ØnZ}}{2p}$$

$$\text{E} = \text{ØnZ} \qquad \text{or} \qquad \text{E} = \text{ØnZ}$$

Ø= 0.02 weber/pole,

n = speed of the generator in rev/sec = $\frac{speed\ in\ rev/min}{60}$

n = speed of the generator in rev/sec = $\frac{600}{60}$ = 10 rev/sec

Z = number of conductors = 1600 conductors

Therefore,

E = 0.02 x 10 x 1600

$E = 320$ Volts

2. e.m.f generated if the armature is wave wound

   Since,

$$E = \frac{P\emptyset nZ}{C} \quad \text{or} \quad E = \frac{2p\emptyset nZ}{c}$$

$$E = \frac{P\emptyset nZ}{2} \quad \text{or} \quad E = \frac{2p\emptyset nZ}{2}$$

$$E = \frac{4x\ 0.02\ x\ 10\ x\ 1600}{2} \quad \text{or} \quad E = \frac{2x2\ x\ 0.02\ x\ 10\ x1600}{2}$$

$E = 640$ Volts

A synchronous machine is illustrated in the figure below:

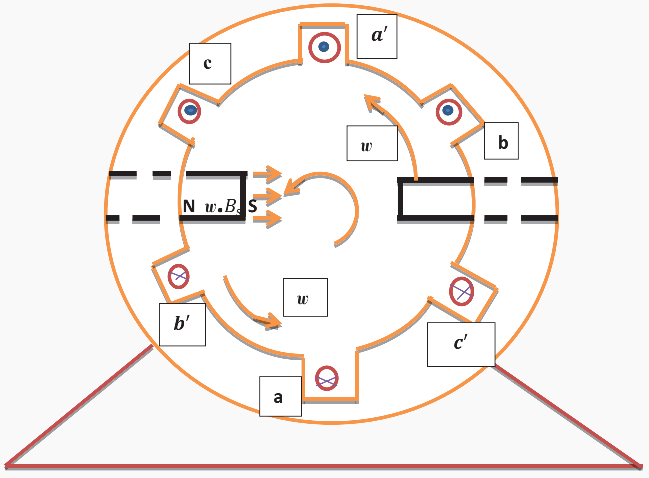


Figure 2.11 : A synchronous machine

Torque is maximum when the angle γ between the rotor and stator poles is 90◦.Also, as we can see from the cross section diagram that in a DC machine the armature is usually on the rotor, and the field winding is on the stator.

During the normal operation of AC motors or generators, there are two magnetic fields.

1. The magnetic fieldB_R from the rotor
2. The magnetic fieldB_S from the stator
3. A rotating torque will be produced in the machine due to the interaction of the two magnetic fields
4. Therefore, the induced torque T_{ind} = k$B_R B_s$ sin ϒ

5. Where Υ = torqueangle betweenB_R and B_S
   The net magnetic field B_{net} in the machine can be obtained as follows:

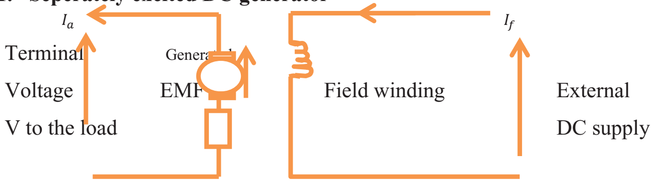
$$B_{net} = B_R + B_S$$

6. Also, the induced torque T_{ind} = k$B_R B_{net}$ sin δ

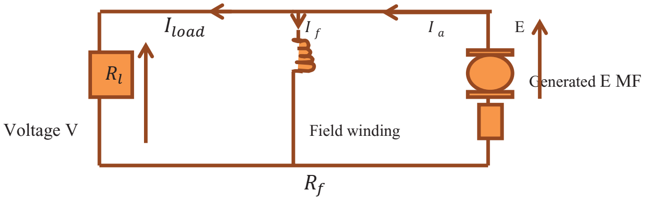
   Where, δ = the load angle

CONFIGURATION OF A DC MACHINE

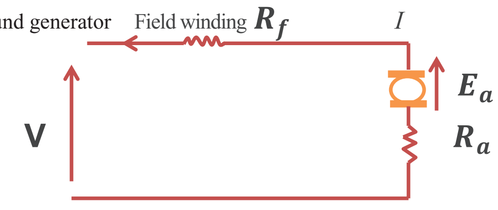
1. **Seperately excited DC generator**



2. Shunt wound generator

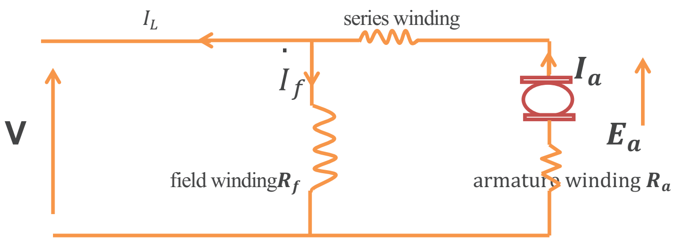


3 Series wound generator



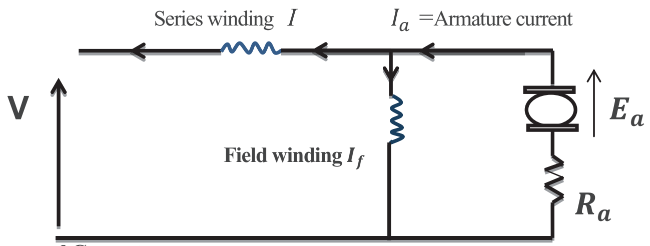
4

Characteristics of DC compound Generator



Long Shunt Compound Generator

Characteristics of DC compound Generator



Short- **Shunt Compound Generator**

Figure 2.12 : showing the Configurations of DC Machines

1. In DC machines, the field excitation that provides the magnetizing current is occasionally provided by an external source, in which case the machine is said to be separately excited figure a above
2. When the field excitation is derived from the armature voltage, the machine is said to be self-excited.
3. The self-excited configuration does not require the use of a separate source for the field excitation and is therefore frequently preferred. If a machine is in the separately excited configuration, an additional source V_f is required.
4. In the self-excited case, a good method that can be used to provide the field excitation is to connect the field in parallel with the armature; since the field winding typically has significantly higher resistance than the armature circuit remember that it is the armature that carries the load current, this will not draw excessive current from the armature.
5. Also, a series resistor can be added to the field circuit to provide the means for adjusting the field current independent of the armature voltage. This configuration is called a shunt-connected machine and is represented in Figure b.
6. Another method for self-exciting a DC machine is connecting the field in series with the armature, leading to the series-connected machine, shown in figure c above; in this case, the field winding will support the entire armature current, and thus the field coil must have low resistance and therefore relatively few turns. This configuration is rarely

used for generators since the generated voltage and the load voltage must always differ by the voltage drop across the field coil, which varies with the load current.

7. Therefore, a series generator would have poor (large) regulation.

   However, series-connected motors are commonly used in applications not more than about 1 kW output, or if we are talking about bigger motors – they are used for electric locomotives.

8. The third type of DC machine is the compound-connected machine, which consists of a combination of the shunt and series configurations. Figure d and e represent the two types of connections, called the short shunt and the long shunt, respectively.
9. Each of these configurations may be connected so that the series part of the field adds to the shunt part (cumulative compounding) or so that it subtracts (differential compounding)

CHAPTER THREE

PREFORMANCE AND METHOD OF SPEED CONTROL OF DC MACHINE

Performance of DC Generator

As we have said earlier, DC generator can be classified based on the method of their excitation.

These classifications are:

5 Separately-excited generators

6 Self-excited generators

1. **SEPARATELY-EXCITED GENERATORS**: This is when the field winding is connected to a different source of power supply other than the armature as shown in the figure below:

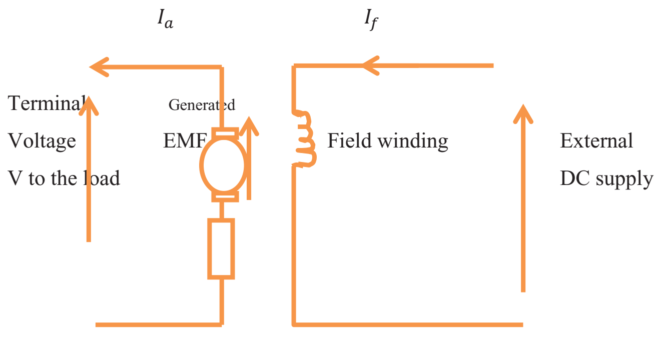


Figure 3.1 : Separately-Excited Generators

When a load is connected across the armature terminals, a load current, I_a will flow through the system.

The EMF E generated will fall to V (the terminal voltage) due to the armature resistance, R_a. Then,

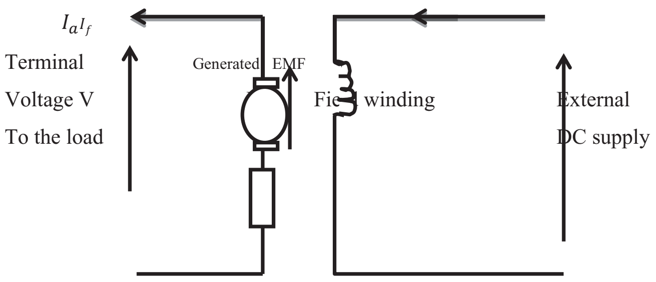
$$V = E - I_a R_a$$

$$E = V + I_a R_a$$

Example 1

A generator develops an E.M.F of 220V. Its armature current at no load is 32A on load. Determine the terminal voltage when the armature resistance is 0.28 A

Solution



In this generator

$$V = E - I_a R_a$$

V = terminal voltage to be determined in this question

E = Generated E.M.F, E = 220 V

I_a = Armature current at no load = 32 A

R_a = Armature resistance = 0.28 A

Hence,

$$V = E - I_a R_a$$

$$V = 220 - 32 \times 0.28$$

$$V = 220 - 8.96$$

$$= 211.04 \text{ Volts}$$

Example 2

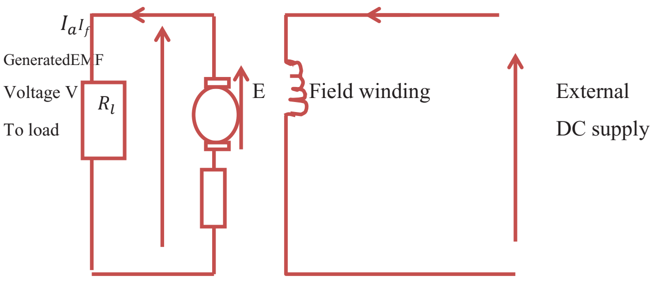
A load of 56 Ω is connected to a generator, then a current of 12A flows. If the armature resistance is 1.3 Ω

Determine the

a. Terminal voltage of the generator
b. Generated E.M.F

Solution

i.



It is clear that $E = V + I_a R_a$ and

$$V = E - I_a R_a$$

It is also clear that Terminal Voltage $V = I_a R_l$

Therefore, $V = 12 \times 56$

$$V = 672 \text{ Volts}$$

ii. To calculate the generated voltage:

Since,

$$E = V + I_a R_a$$

$$E = 672 + 12x1.3$$

$$E = 687.6 \text{ Volts}$$

Example 3

A separately-excited DC generator develops a no-load E.M.F of 200 V at an armature speed of 22 rev/s and a flux per pole of 0.15Wb.

Determine the Generated E.M.F when:

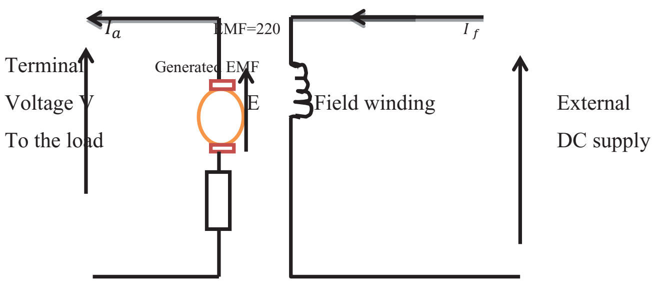
(a) the speed increases to 27 rev/s and the pole flux remains unchanged,

(b) the speed remains at 22 rev/s and the pole flux is decreased to 0.12Wb,

(c) the speed increases to 24 rev/s and the pole flux is decreased to 0.10Wb.

Solution

At no load, we have

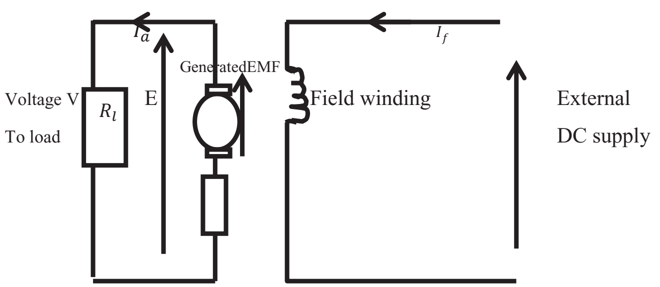


Flux per pole = Ø = 0.15 Wb

Let the mechanical speed of the machine in revolution per second be = n_m

armature speed in rev/sec = n_m = 22 rev/s

When load is now connected, we have:



Let us go through this explanation again

EMF GENERATED IN A DC GENERATOR

When a load is connected across the armature of terminals, a load current, I_a will flow.

The EMF E generated will fall to V (the terminal voltage) due to the armature resistance, R_a.

$$V = E - I_a R_a$$

$$E = V + I_a R_a$$

The E.M.F Generated by the armature is equal to the e.m.f generated by one of the parallel paths.

During one revolution, each conductor will pass through P poles.

The total magnetic flux in the core per pole is Ø.

1 pole = Ø weber

Therefore, each conductor will cut a total flux of P xØ weber per revolution

1 pole = Ø weber

P poles = ?

Then, total number of flux in one revolution =P Ø weber

In one revolution, total flux is P Ø weber

The armature makes n revolutions in one second.

if one revolution = P Ø weber

Then, n revolutions = x *weber*

Total flux in one second= x = P Ø n weber

It is important for you to know that 1 volt = 1weber per second

Hence, the e.m.f E generated per conductor can now be calculated as follows:

E = P Ø n volts   P = number of poles

Also, E = 2pØ n volts p = number of pairs of poles, P = 2p

There are 2 brushes pressed against the commutator in a DC machine

The number of conductors in series in each path = $\frac{Z}{c}$

Where,

Z = number of conductor in series

c = number of parallel paths

c = 2, for a wave winding

c = 2p or P, in a lap winding

Hence, the total e.m.f generated = average e.m.f generated /conductor x number of conductor in series in each path

$$E = P\,Ø\,n\,x\frac{Z}{c} \quad \text{volts}$$

$$E = \frac{PØnZ}{C} \quad \text{or} \quad E = \frac{2pØnZ}{C}$$

Let the number of pole P, number of pairs of pole p, number of conductors in series Z and the number of parallel paths gives a constant value k. Then,

$$E = kØn \ , \quad k = \frac{PZ}{C}$$

or $$E = kØn \ , \quad k = \frac{2pZ}{C}$$

Since

$$E = kØn \ , \quad k = \frac{2pZ}{C} p = number\ of\ pairs\ of\ pole$$

At no load

We can easily see hat E = 200V,*k is constant,* Flux per pole = Ø and Ø = 0.15 Weber, armature speed in rev/sec = n_m = 22 rev/s

Therefore, it is now clear from

$E_1 = kØ_1 n_1$

220 =k x 0.15x 22………………………………………….. (1)

1. Later, the speed increases to 27 rev/s and the pole flux remains unchanged

Since

$E_2 = k\text{Ø}_2 n_2$

$E_2 = k$ x 0.15x 27 ………………………………………… (2)

Comparing equations 1 and 2 will make this question a simple one.

$$\frac{E_1}{E_2} = \frac{k\text{Ø}_1 n_1}{k\text{Ø}_2 n_2}$$

Hence

$$\frac{200}{E_2} = \frac{K\ X\ 0.15\ X\ 22}{K\ X\ 0.15\ X27}$$

Since k is constant

Then $E_2 = \frac{200X\ 0.15X27}{0.15X22}$

$E_2 = 245.455$ Volts

2. At no load

We have seen that E = 200V,*k is constant,* Flux per pole = Ø and Ø = 0.15 Weber, armature speed in rev/sec = n_m = 22 rev/s

Therefore, it is now clear from

$E_1 = k\text{Ø}_1 n_1$

200 = k x 0.15x 22…………………………………………... (1)

Later, the speed remains at 22 rev/s and the pole flux is decreased to 0.12 Wb,

Since

$E_3 = k\text{Ø}_3 n_3$

$E_3 = k$ x 0.12x 22 …………………………………………(2)

Comparing equations 1 and 2 will make this question a simple one.

$$\frac{E_1}{E_3} = \frac{k\text{Ø}_1 n_1}{k\text{Ø}_3 n_3}$$

Hence

$$\frac{200}{E_3} = \frac{K\ X\ 0.15\ X\ 22}{K\ X\ 0.12\ X22}$$

Since *k* is constant

Then $E_3 = \frac{200X\ 0.12X22}{0.15X22}$

Hence,

E_2= 160 Volts

c. At no load

We have seen that E = 200V, *k is constant,* Flux per pole = Ø and Ø = 0.15 Weber, armature speed in rev/sec = n_m = 22 rev/s

Therefore, it is now clear from

$E_1 = k\text{Ø}_1 n_1$

200 = k x 0.15x 22………………………………………... (1)

Later, the speed increases to 24 rev/s and the pole flux is decreased to 0.10Wb.

Since

$E_4 = k\text{Ø}_4 n_4$

$E_4 = k$ x 0.10x 24 ………………………………………… (2)

Comparing equations 1 and 2 will make this question a simple one.

$$\frac{E_1}{E_4} = \frac{k\text{Ø}_1 n_1}{k\text{Ø}_2 n_2}$$

Hence

$$\frac{200}{E_4} = \frac{K\ X\ 0.15\ X\ 22}{K\ X\ 0.10\ X24}$$

Since *k* is constant

Then $E_4 = \frac{200X\ 0.10X24}{0.15X22}$

Hence,

E_4 = 145.455 Volts

Characteristics of DC Generators

Generally, following three characteristics of DC generators are taken into considerations:

(i) Open Circuit Characteristic (O.C.C.),

(ii) Internal or Total Characteristic and

(iii) External Characteristic.

1. Open Circuit Characteristic (O.C.C.) (E_0/I_f) of all DC generators

Open circuit characteristic can also be called magnetic characteristic or no-load saturation characteristic. This characteristic shows the relation between generated emf at no load (E_o) and the field current (I_f) at a given fixed speed.

The Open Circuit Characteristic (O.C.C.) curve is just the magnetization curve and it is practically similar for all type of generators. The data for O.C.C. curve is obtained by operating the generator at no load and keeping it at a constant speed. Field current will then increase gradually. The corresponding terminal voltage will be recorded. The figure below shows the connection arrangement to obtain O.C.C. curve.

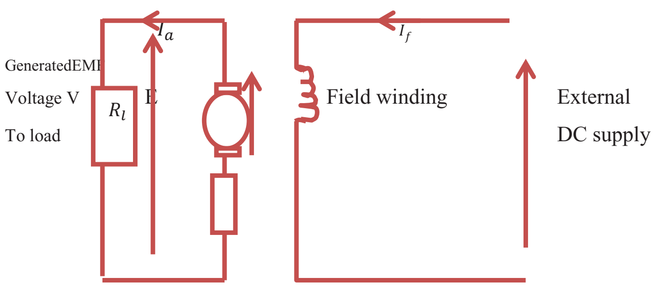


Figure 3.2: Separately-Excited generator, load current I_l is equal to the armature current I_a

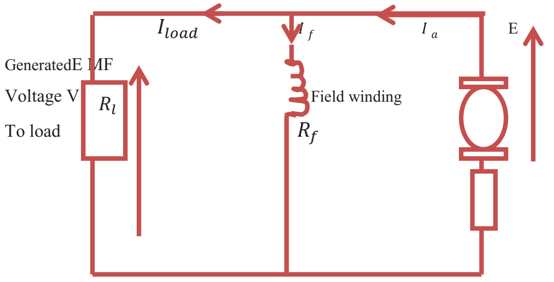


Figure 3.3 : Shunt wound generator

Here,

$I_a = I_f + I_l$

Terminal voltage V = E_g - $I_a R_a$

Then, terminal voltage V = E_g - (I_{sh}+ I_l) R_a

Please take note

That for shunt or series excited generators; the field winding is disconnected from the machine and connected across an external supply as shown below:

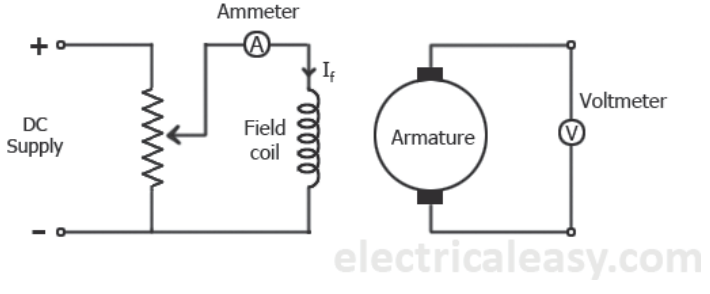


Figure 3.4 : Separately excited generator, **Reference: www.electricaleasy.com**

From the e.m.f equation of dc generator,

The total e.m.f generated E or E_g = average e.m.f generated /conductor x number of conductor in series in each path

$$E = P\,\emptyset\,n \times \frac{Z}{C} \quad \text{volts}$$

$$E = \frac{P\emptyset nZ}{C} \quad \text{or} \quad E = \frac{2p\emptyset nZ}{C}$$

Let the number of pole P, number of pairs of pole p, number of conductors in series Z and the number of parallel paths gives a constant value k. Then,

$$E = k\emptyset n \;, \qquad k = \frac{PZ}{C}$$

or

$$E = k\emptyset n \;, \qquad k = \frac{2pZ}{C}$$

We have seen that

$$E = k\phi n$$

Hence, the generated e.m.f E should be directly proportional to field flux. Therefore, it is also directly proportional to the field current, I_f. However, even when the field current is zero, some amount of e.m.f will still be generated. This initially induced emf is due to the fact that there are some residual magnetism in the field poles. This is represented by OA in the figure below. Due to this residual magnetism, a small initial e.m.f is induced in the armature.

This initially induced e.m.f aids the existing residual flux, and hence, increasing the overall field flux. This consequently increases the induced emf. Thus, O.C.C. follows a straight line. However, as the flux density increases, the poles get saturated and the ϕ becomes practically constant. Thus, even we increase the I_f further, ϕ remains constant and hence, E also remains constant. Hence, the O.C.C. curve looks like the B-H characteristic.

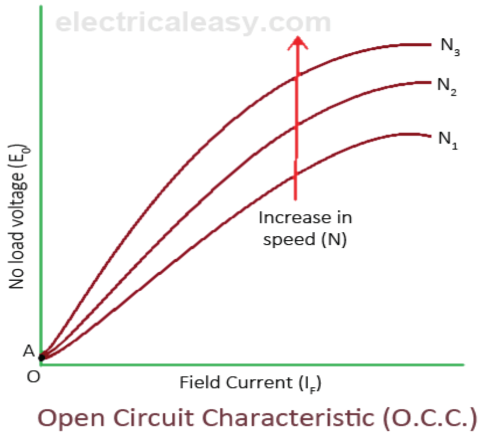


Figure 3.5 : T**ypical no-load saturation curve or open circuit characteristics for all types of DC generators. Reference: www.electricaleasy.com**

2. Internal Characteristic of Shunt Wound DC Generator

The internal characteristic curve represents the relationship between the generated voltage E_O and the load current I_L. When the generator is loaded then the generated voltage is decreased due to armature reaction. So, generated voltage will be lower than the emf generated at no load. Hence, in the figure 2.19 below, AD curve shows the no load voltage curve and AB is the internal characteristic curve.

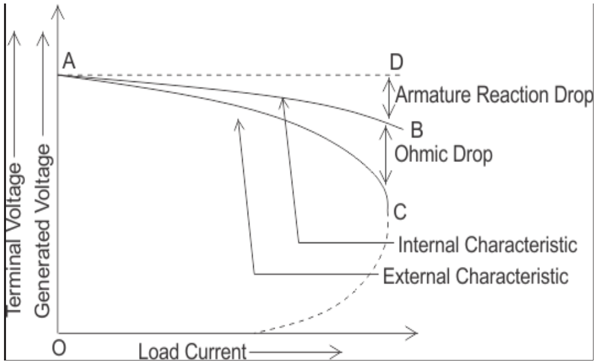


Figure 3.6: shows the no load voltage curve and AB is the internal characteristic curve
Reference: www.electricaleasy.com

3. External Characteristic or load Characteristic of Shunt Wound DC Generator **Reference: www.electricaleasy.com**

External characteristic as it name implies is the same as terminal voltage –load current characteristics (V/I_L)

Hence, an external characteristic curve shows the relation between terminal voltage (V) and the load current (I_L) i.e. (V/I_L). Terminal voltage V is less than the generated e.m.f E due to voltage drop in the armature circuit. Therefore, external characteristic curve lies below the internal characteristic curve. External characteristics are very important to determine the suitability of a generator for a given purpose. Therefore, this type of characteristic is sometimes called **performance characteristic** or **load characteristic**.

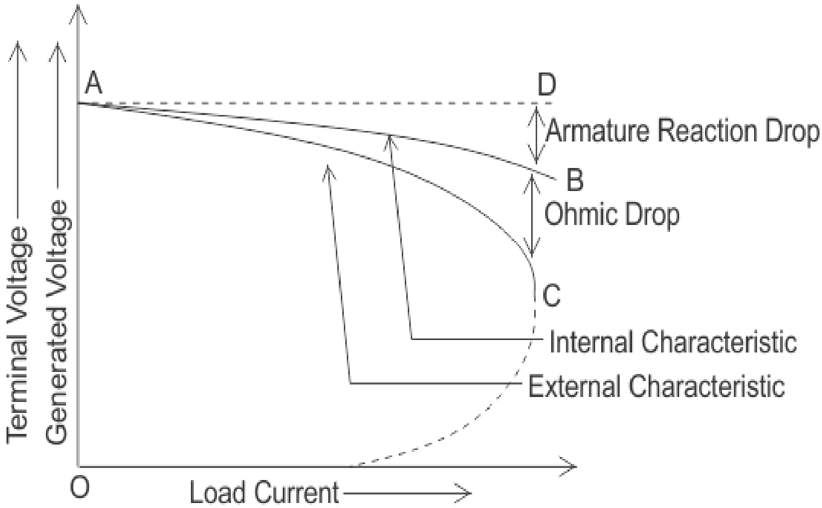


Figure 3.7: Internal and External Characteristics **Reference: www.electricaleasy.com**

External Characteristics

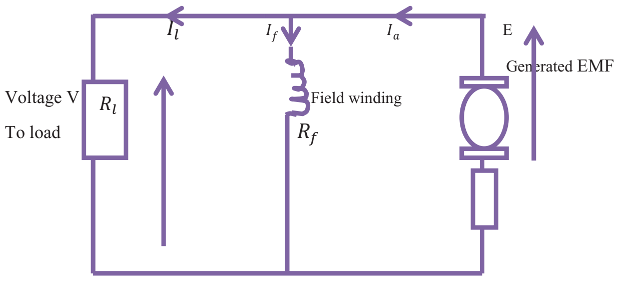


Figure 3.8: Shunt wound generator

$$I_a = I_f + I_l$$

Terminal voltage V = E_g - $I_a R_a$

Then, terminal voltage V = E_g - $(I_{sh} + I_l)\ R_a$ $\qquad I_f = I_{sh}$

In the figure below, AC curve represents the external characteristic of the shunt wound DC generator. It shows the variation of terminal voltage with the load current, I_L.

Ohmic drop due to armature resistance gives lesser terminal voltage than the generated voltage. That is why the curve lies below the internal characteristic curve.

Terminal voltage V = E_g - $I_a R_a$

Terminal voltage V = E_g - $(I_{sh}+ I_l)$ R_a

The terminal voltage can always be maintained constant by adjusting the of the load terminal.

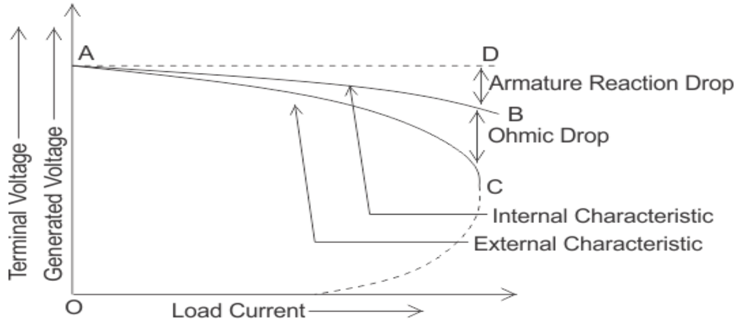


Figure 3.9 : Internal and External Characteristics **Reference: www.electricaleasy.com**

When the load resistance of a shunt wound DC generator is decreased, then load current of the generator increased as shown in above figures. But the load current can be increased to a certain limit with (up to point C) the decrease of load resistance. Beyond this point, it shows a reversal in the characteristic. Any decrease of load resistance, results in current reduction. Consequently, the external characteristic curve turns back as shown in the dotted line. Ultimately the terminal voltage becomes zero. Though there is some voltage due to residual magnetism.

We know that, Terminal voltage

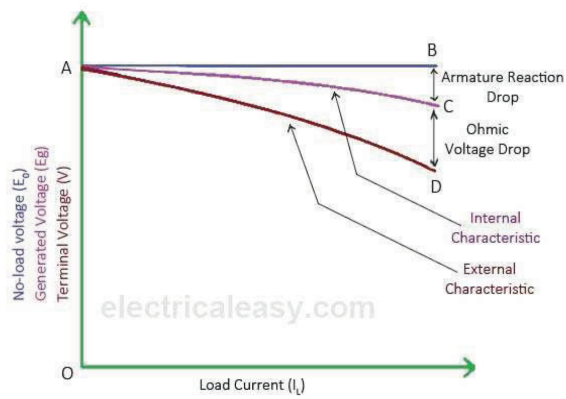
$$V = E_g - (I_{sh}+ I_l)\ R_a$$

Now, when I_L increased, then terminal voltage decreased. After certain limit, due to heavy load current and increased ohmic drop, the terminal voltage is reduced drastically. This drastic reduction of terminal voltage across the load, results the drop in the load current although at that time load is high or load resistance is low.

That is why the load resistance of the machine must be maintained properly. The point in which the machine gives maximum current output is called breakdown point (point C in the picture)

Hence, beyond this limit any further decrease in load resistance results in decreasing load current. Consequently, the external characteristic curve turns back as shown by dotted line in the figure above

Figure3.10: Characteristics of Separately Excited Generator, Reference: www.electricaleasy.com



Characteristics of separately excited DC generator

Characteristics of Separately Excited DC Generator **Reference: www.electricaleasy.com**

1. If there is no armature reaction and armature voltage drop, the voltage will remain constant for any load current. ***Therefore, the straight- line AB in the figure above represents the no-load voltage against the load current I_L.***
2. Because of the demagnetizing ***effects of armature reaction, the on-load generated e.m.f is lower than the no-load voltage. The curve AC represents the graph of the on-load generated emf E against the load current I_L*** i.e. internal characteristic (as armature current I_a = load current I_L for a separately excited dc generator).

3. Also, ***the terminal voltage is lesser due to ohmic drop occurring in the armature and brushes. The curve AD represents the terminal voltage vs. load current i.e. external characteristic.***

Characteristics of DC Series Generator

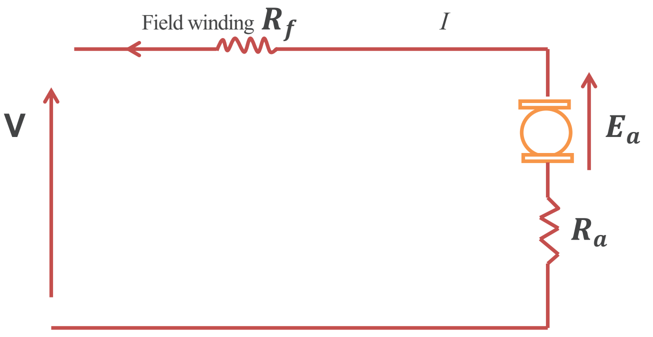


Figure3.11: Series-Wound Generator

It is clear that the same current I will flow through the armature winding and the field winding.

The load characteristic is the graph of the terminal voltage against the current.

The e.m.f generated is directly proportional to Ø ω

$$\text{Where } \omega = 2\pi n$$

The field resistance and the armature resistance are very small, therefore, the terminal voltage is almost equal to the generated e.m.f as shown below:

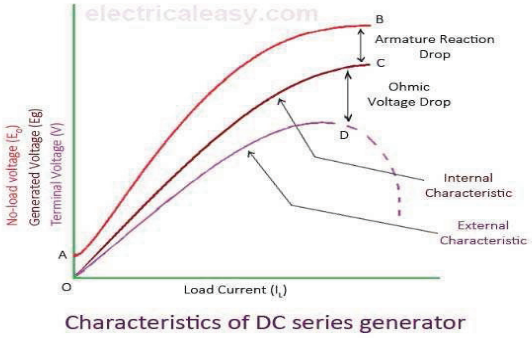


Figure3.12: DC Series Generator Reference: www.electricaleasy.com

1. The curve AB in the figure above is identical to open circuit characteristic (O.C.C.) curve. This is because in DC series generators, field winding is connected in series with armature and load.
2. Therefore, the load current is similar to field current i.e. I_L=I_f.
3. The curve OC and OD represent internal and external characteristic respectively.
4. In the DC series generator, terminal voltage increases with the load current. This is because, as the load current increases, field current also increases.
5. However, beyond a certain limit, terminal voltage begins to decrease with increase in load. This is because of the excessive demagnetizing effects of the armature reaction.

Characteristics of DC Compound Generator

The figures below show the two types of compound –wound generator.

The first diagram is the long shunt Compound Generator. The second diagram is the long shunt Compound Generator.

Characteristics of DC compound Generator

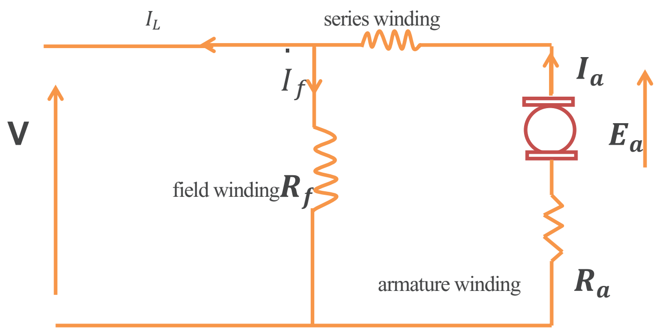


Figure3.13 : Long Shunt Compound Generator

In long shunt compound generator,

$$I_a = \quad I \ + I_f$$

Characteristics of DC compound Generator

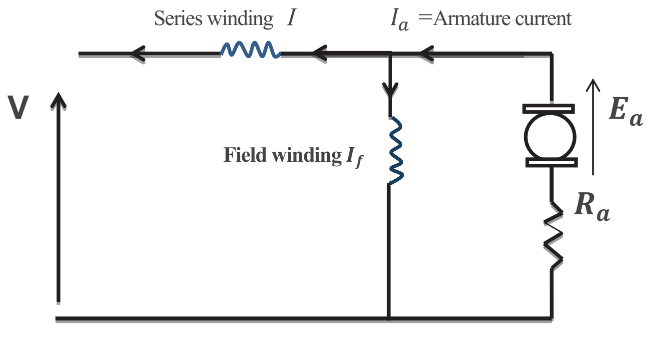


Figure 3.14 : Short- **Shunt Compound Generator**

In Short-shunt compound generator,

$$I_a = \quad I \ + I_f$$

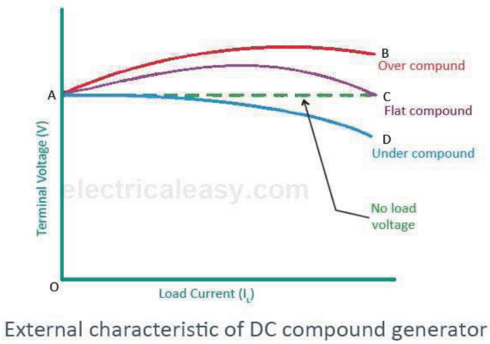


Figure 3.15: the external characteristics of DC compound generators.

Reference: www.electricaleasy.com

1. If series winding amp-turns are adjusted so that, increase in load current causes increase in terminal voltage then the generator is called to be over compounded.

2. The external characteristic for over compounded generator is shown by the curve AB in above figure.

3. If series winding amp-turns are adjusted so that, the terminal voltage remains constant even the load current is increased, then the generator is called to be flat compounded or level compounded generator.

4. The external characteristic for a flat compounded or level compounded generator is shown by the curveAC

5. If the series winding has lesser number of turns than that would be required to be flat compounded, then the generator is said to be under compounded generator. The external characteristics for an under compounded generator are shown by the curve AD

METHOD OF SPEED CONTROL OF DC MACHINE

Speed control is Intentional speed variation carried out manually or automatically to control the speed of DC motors or the intentional speed variation techniques carried out to vary the speed of DC machines

SPEED CONTROL OF DC MOTOR

We have seen that in a DC motor, if the applied voltage is V. When the DC motor rotates, an e.m.f will be induced in the armature windings. This induced e.m.f is E. From Lenz law, the induced e.m.f E will oppose the motion of the supply voltage that produces it.

This e.m.f, E is known as back e.m.f. It is less than the applied voltage V due to the armature reaction

$$V = E + I_a R_a$$

$$E = V - I_a R_a \qquad \text{...........................(1)}$$

Also, we have seen that, the back e.m.f is equal to E

$$E = k\emptyset n \quad , \qquad k = \frac{PZ}{C}$$

or

$$E = k\emptyset n \quad , \qquad k = \frac{2pZ}{C}$$

In an electric motor, the armature makes *n* revolutions in one second. So, *n* is in revolution per second. From elementary that means n is the same thing as frequency f, in rev/sec or hertz

Hence,

If the mechanical speed of the generator is ω in radian/sec

Then $\omega = 2\pi n$

$$n = \frac{\omega}{2\pi}$$

Since,

$$E = \frac{P\emptyset nZ}{C} \quad \text{or} \quad E = \frac{2p\emptyset nZ}{C} \quad \text{n is in rev/sec}$$

Then,

$$E = \frac{P\emptyset\omega Z}{2\pi C} \text{ or } E = \frac{2p\emptyset\omega Z}{2\pi C}$$ **ω is in radian/sec**

$$E = K_a \emptyset \omega \quad \text{……………………..(2)}$$

Where, armature constant K_a will be

$$K_a = \frac{PZ}{2\pi C}$$ P = number of poles

$$K_a = \frac{pZ}{\pi C}$$ p = number of pairs of poles

And P = 2 p

Where,

K_a= Armature constant = $\frac{PZ}{2\pi a}$

P = No. of poles,

Z = Total no. of armature conductor,

c = No. of parallel paths

From equation 1 and 2, it can be seen that

$$E = V - I_a R_a$$

$$E = K_a \emptyset \omega$$

Then,

$$K_a \emptyset \omega = V - I_a R_a$$

$$\omega = \frac{V - I_a R_a}{K_a \emptyset} \quad \text{…………………………………….(3)}$$

From this equation, it can be confirmed that for DC motor,

THERE ARE BASICALLY 3 METHODS OF SPEED CONTROL

They include:

1. Variation of resistance in armature circuit R_a
2. Variation of field flux Ø
3. Variation of armature terminal voltage V

SPEED CONTROL OF D.C. MOTORS

Shunt-wound motors

We have seen that the speed of a shunt-wound d.c. motor, n or ω, is proportional to

$$\omega = \frac{V - I_a R_a}{K_a \text{Ø}}$$

Hence,

1. The speed is varied either by varying the valueof flux, Ø, or

2. By varying the value of R_a. This is achieved byusing a variable resistor which we called shunt field regulator in series with the field winding, as shown in the figure below:

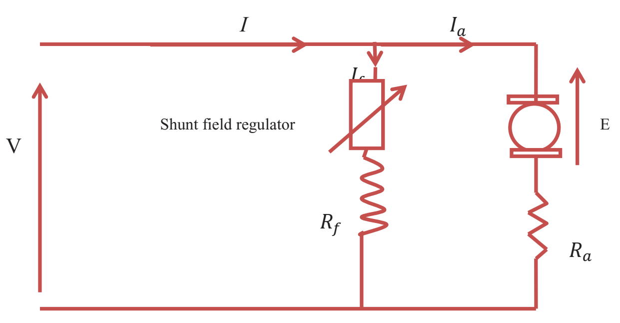


Figure 3.16 : shunt field regulator in series with the field winding

As the value of resistance of the shunt field regulator is increased,

1. the value of the field current, I_f will decrease
2. the value of the armature current, I_a will increase
3. as the value of the field current, I_f decreases, the value of field flux, Øwil decrease

4. Hence, there will be an increase in the speed of the motor. Note that only speeds abovethat given without ashunt field regulator can be obtained by this method. Speeds below thosegiven by

$$\omega = \frac{V - I_a R_a}{K_a \text{Ø}}$$

are obtained by increasing the resistance in the armature circuit as shown below:

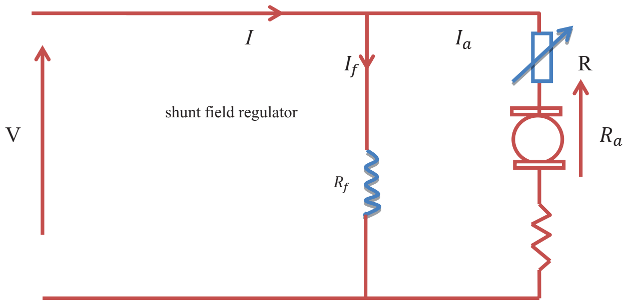


Figure 3.17 : shunt field regulator in series with the field winding

$$\omega = \frac{V - I_a (R_a + R)}{K_a \text{Ø}}$$

Since resistor R is in series with the armature,

1. It carries the full armature current.
2. Hence, results in a large power loss in large motors where a considerable speed reduction is required for long periods.

These methods of speed control are demonstrated in the example below

Example 4

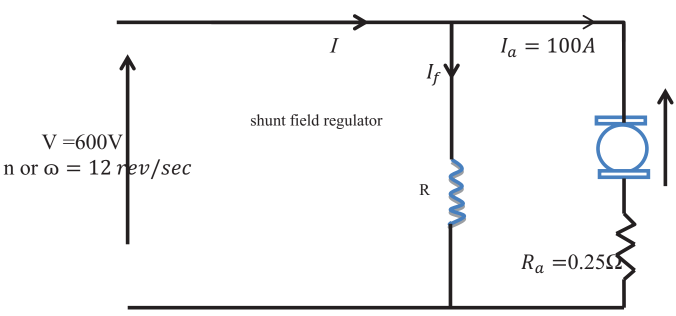
A 600 V shunt motor runs at its normal speed of 12 rev/sec when the armature current is 100A. The armature resistance is 0.25Ω. Determine

(a) the back e.m.f

(b) the speed when the armature current is 68 A and a resistance of 0.4Ω is connected in series with the armature, the shunt field remaining constant.

(c) Determine the speed when the current is 68 A and the shunt field is reduced to 76% of its normal value by increasing resistance in the field circuit

Solution:



We have seen that in a DC motor, if the applied voltage is V. When the DC motor rotates, an e.m.f will be induced in the armature windings. This induced e.m.f is E. From Lenz law, the induced e.m.f E will oppose the motion of the supply voltage that produces it.

This e.m.f E is known as back e.m.f . It is less than the applied voltage V due to the armature reaction

$$V = E + I_a R_a$$

$$E = V - I_a R_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Also, we have seen that, the back e.m.f is equal to E

$$E = k\emptyset n \ , \qquad k = \frac{PZ}{C} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

or $\quad E = k\emptyset n \quad , \qquad k = \frac{2pZ}{c}$ ……………………..(3)

P = number of poles, $\quad p$ = Number of pairs of poles

(a)
From equation 1

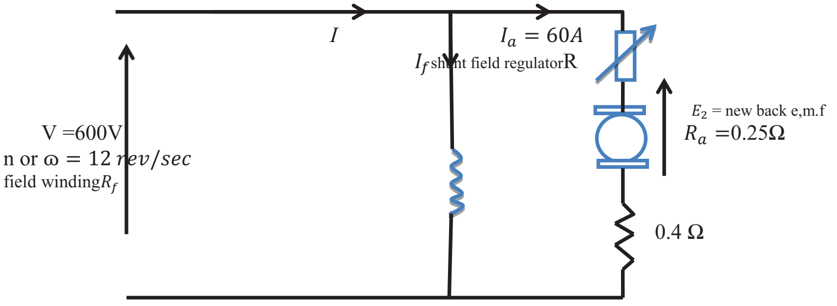
$$E = V - I_a R_a$$

$$E = 600 - 100 \times 0.25$$

$$E = 600 - 25$$

E = 575 Volts

b.
Now, when the armature current is 68 A and a resistance of 0.4 Ω is connected in series with the armature, the shunt field remaining constant as shown below:



$E = V - I_a R_a$ $\quad$ will change to

$$E_2 = V - I_a(R_a + R)$$

Hence, $\quad E_2 = 600 - 68\,(0.25 + 0.4)$

$$E_2 = 600 - 68\,(0.65)$$

$$E_2 = 600 - 44.2$$

E_2 = 555.8 Volts

From equation 2

Back e.m.f, E $= k\text{Ø}n$

Since the shunt field remains constant, then:

$$E_1 = k\text{Ø}n_1$$

$$E_2 = k\text{Ø}n_2$$

575 $= k\text{Ø} \text{ x } 12$……………………………….(4)

555.8 = $k\text{Ø} \text{ x } n_2$……………………………….(5)

Dividing equation 4v by equation 5

$$n_2 = \frac{555.8\ x12}{575}$$

n_2= 11.599 rev/sec

SERIES-WOUND MOTORS

The speed control of series-wound motors is achieved using either

(a) field resistance, or

(b) armature resistance techniques.

The speed of a d.c. series-wound motor is given by:

$$E = k\emptyset n \qquad \text{and}$$

$$E = V - I_a R_a$$

$$k\emptyset n = V - I_a R_a$$

$$n = \frac{V - I_a R_a}{k\emptyset}$$

Where:

1. K is a constant,
2. V is the terminal voltage,
3. R is the combined resistance of the armature and series field and
4. $\emptyset$ is the flux
5. Therefore, a reduction in flux results in an increase in speed.

(a) This is achieved by putting a variable resistance in parallel with the field winding. This will reduce the field current. Hence, the flux will reduce, for a given value of supply current and the speed will increase. This variable resistor connected in parallel with the series-wound field to control speed is called a diverter. This is shown in Figure2.31 below. Speeds above those given with no diverter are obtained by this method.

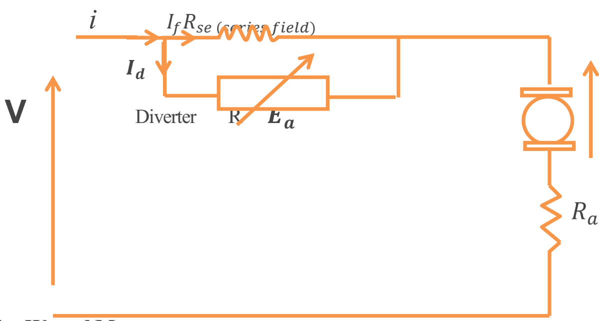


Figure 3.18 : Series-Wound Motor

(b) While speeds below normal are obtained by connecting a variable resistor in series with the field winding and armature circuit, as shown below:

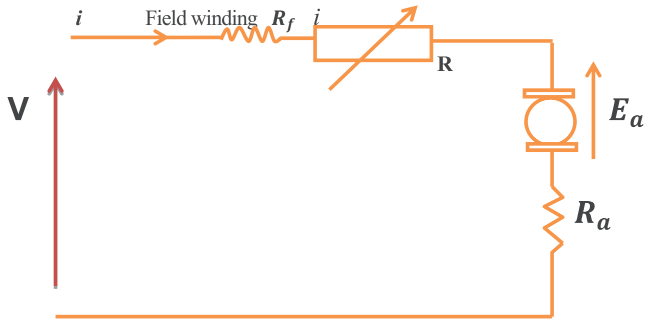


Figure 3.19 : Series-Wound Motor

1. Increase in the value will reduce the speed of the DC motor.
2. Since the additional resistor R carries the full supply current,
3. a large power loss is associated with large motorsin which a considerable speed reduction is required for long periods.

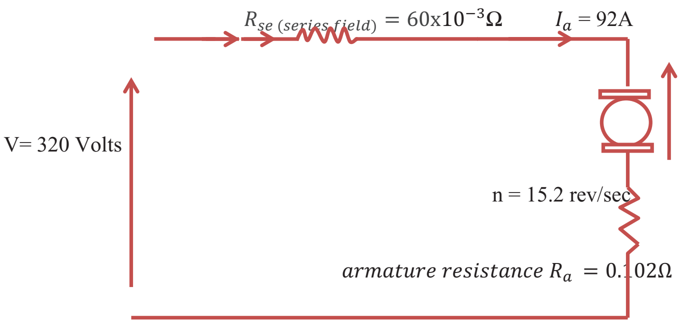
These methods of speed control are demonstrated in the example below

Example 5

On full-load a 320 V series motor takes 92 A and runs at 15.2 rev/sec. The armature resistance is 0.102Ω and the series winding resistance is 60 mΩ. Determine the speed when developing full load torque but with a 0.24Ω diverter in parallel with the field winding. Assume that the flux is proportional to the field current.

Solution:

Without the diverter the DC machine will be in this form:



In the circuit above, I_a is = $92\ A$ and $Ø$ *is proportional to the field current* which in this circuit is the same as 92A

Hence,

$$Ø\ \alpha I_f$$

ThenT which is = $kØI_a$ will become T $= kI_f I_a$ ……………………………(1)

$$T_1 = k92\ x92$$

In a DC motor, if the applied voltage is V. When the DC motor rotates, an e.m.f will be induced in the armature windings. This induced e.m.f is E. From Lenz law, the induced e.m.fE will oppose the motion of the supply voltage that produces it.

This e.m.f E is known as back e.m.f. It is less than the applied voltage V due to the armature reaction

$$V = E + I_a R_a$$

$$E = V - I_a R_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Also, we have seen that, the back e.m.f is equal to E

$$E = k Ø n \ , \qquad k = \frac{PZ}{C} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

or $$E = k Ø n \ , \qquad k = \frac{2pZ}{C} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

P = number of poles, p = Number of pairs of poles

Hence, without a diverter

$$E = V - I_a R \quad R = R_a + R_f$$

$$E = 320 - 92x(R_a + R_f)$$

$$E = 320 - 92x\ (0.102 + 0.06)$$

$$E = 320 - 92x\ 0.162$$

$$E = 320 - 14.904$$

$$E = 305.096 \text{ Volts}$$

Therefore, without a diverter

$$T_1 = k Ø I_a$$

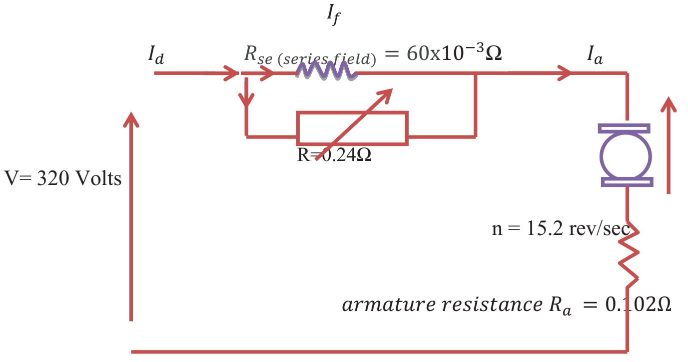
$$T = k Ø \text{ x } 92$$

The assumption is that the flux is proportional to the field current i.e. $Ø\ \alpha\ I_f$ *and* $I_f = 92A$

$$Ø\ \alpha\ I_f \textit{ will mean } Ø\ \alpha 92\ A$$

Torque, 'T' that was $= k Ø$ x 92 will now be T = k x92 x 92

Now, with the 0.24Ω diverter in parallel with *series field resistance Rse*, the DC machine will be in this form



The equivalent resistance = R_{eq}

$$R_{eq} = \frac{0.24x0.06}{0.24+0.06}$$

$$R_{eq} = \frac{0.0144}{0.3} = 0.048\ \Omega$$

You will notice that the equivalent resistance has changed, then the current drawn from the supply will no longer be $I_a = 92\text{A}$

This is because I = $\frac{V}{R\ total}$ if R total changes we know that I will change

So, let the new current drawn be = I_d

Hence, with the diverter

$$E = V - I_a R \qquad R = R_a + R_f,\ I_a is\ now\ I_d$$

$$E = 320 - I_d \text{x} (R_a + R_{eq})$$

$$E = 320 - I_d \text{x} (0.102 + 0.048)$$

$$E = 320 - 0.15\ I_d$$

$the\ new\ back\ e.m.f$, $E = 320 - 0.15\ I_d$

That means we need to calculate I_d $before\ we\ know\ the\ new\ back\ e.m.f$ E

By current division, current in the field winding is

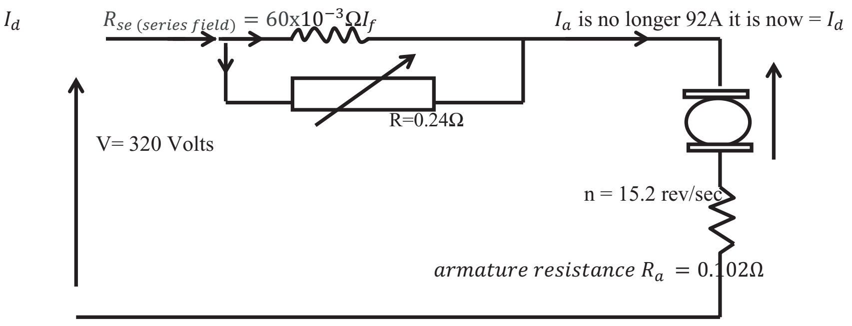
$I_f = \frac{0.24}{0.24+0.06} \times I_d$ that means

$$I_f = 0.8 \text{x} I_d$$

$$I_f = 0.8 I_d$$

We already know that Torque T

$$T = k\text{Ø}I_a$$



The equivalent resistance = R_{eq}

In the circuit above, I_a is now = I_d and *Ø is proportional to the field current* ,

then

$$\text{Ø}\ \alpha I_f$$

Then Torque 'T' which was= $k\text{Ø}I_a$ will become

$$T_2 = kI_f I_d$$

Since, $I_f = \quad 0.8I_d$

$T_2 = kI_fI_d$ will become $T_2 = k0.8I_d\, I_d$ ……………………………..(5)

$$T_1 = kI_aI_a \qquad \text{from equation 1}$$

And I_f $=0.8I_d$

NOTE: The question says "determine the speed when developing full load torque"
That means the torque remain constant $T_1 = T_2$

$$kI_fI_d = kI_aI_a$$

$$k0.8I_dI_d = kI_aI_a$$

$$k0.8I_dI_d = k92\text{x}92$$

$$k0.8{I_d}^2 = k8464$$

$${I_d}^2 = \frac{8464}{0.8} = 10580$$

$I_d = \quad 102.859$ A

Since $\quad E = 320 – I_d \text{x} (0.102 + 0.048)$

E = 320–102.859 x (0.102 + 0.048)

E = 320–102.859 x (0.102 + 0.048)

E = 320- 102.859 x 0.15

E = 320- 15.42885

E = 304.57115Volts

Now, we have seen that, the back e.m.f E = 304.57115 Volts

$$E = k\emptyset n \ , \qquad k = \frac{PZ}{C}$$

or $E = k\emptyset n$, $k = \frac{2pZ}{C}$

Then,

$$E = k\emptyset n = kI_f n_1 \quad \text{When there is no diverter}$$

$E = k\emptyset n = kI_f n_2 = k0.8I_d n_2$ When there is diverter because $I_f = 0.8I_d$

$305.096 = k$ 92 x 15.2 ………………….(1) When there is no diverter

$304.57115 = k$ 0.8 x 102.859 x n_2 …………(2) When there is diverter

Dividing the 2 equations above

$$n = \text{new speed of the motor} = \frac{304.57115 \text{ X } 92 \text{ x } 15.2}{305.096 \text{ X } 0.8 \text{ x } 102.859}$$

$$n = \text{new speed of the motor} = 16.965 \text{ rev/sec}$$

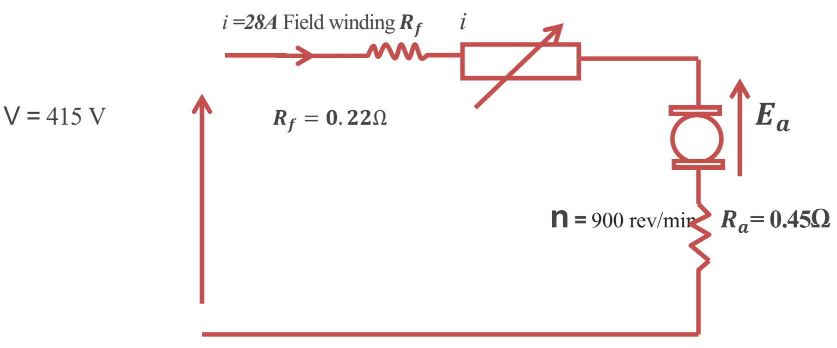
You can see that the speed of the motor has increased from 15.2 rev/s (912 rev/min) to 16.965 rev/s (1017.9 rev/min) by inserting a 0.24 Ω diverter resistance in parallel with the series winding.

Example 6

A series motor runs at 900 rev/minute when the voltage is 415V and the current is 28 A. The series field resistance is 0.22Ω and the armature resistance is 0.45Ω. Determine the resistance of the field regulator to be connected in series with the armature winding to reduce the speed to 700 rev/min with the same current.

Without any resistance or field regulator

Field winding and armature circuit are as shown below:



Series-Wound Motor

Hence, without a diverter

$$E = V - I_a R \qquad R = R_a + R_f$$

$$E = V - I_a \times (R_a + R_f)$$

$$E = 415 - 28 \times (0.45 + 0.22)$$

$$E = 415 - 28 \times (0.67)$$

$$E = 415 - 18.76$$

Therefore the back EMF, E =396.24 V when the speed is 900 rev/min

Now, the back e.m.f needed to produce this speed of 700 rev/min can be obtained easily from

$E = k\phi n$ when the speed is900 rev/min

$E = k\phi n$ when the speed is700 rev/min

$396.24 = k\phi x900$ when the speed is900 rev/min

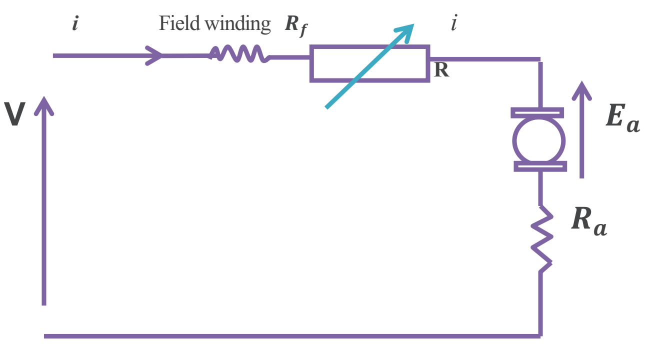
$E = k\phi 700$ when the speed is700 rev/min

Solving the two equation above

$$E = \frac{396.24 \text{ x } 700}{900}$$

E = 308.187 V

Now let us connect the additional resistance by connecting a variable resistor in series with the field winding and armature circuit, as shown below



Series-Wound Motor

Since we know the new speed, we can easily obtain the value of R as shown below:

$E = V - I_a R_a$ will change to

$E_2 = V - I_a(R_a + R_f + R)$

308.187 = 415 - 28 (0.45 +0.22 + R)

308.187 = 415 - 28 (0.67 + R)

$28\ (0.67 + R) = 415 - 308.187$

$28\ (0.67 + R) = 106.813$

$0.67 + R = 3.8148$

$R = 3.8148 - 0.67$

$R = 3.1448\ \Omega$

Thus the addition of a series resistance of 3.1448 Ω has reduced the speed from 900 rev/min to 700 rev/min

MOTOR COOLING

Motors are sometimes classified according to the type of enclosure used in the construction of the motors. This depends on the conditions under which the motor is being used and the degree of ventilation required during the operation of the motor.

The types of protection are:

1. **Screen-protected type: this is used** where ventilation is achieved by fitting a fan internally. This is done through the openings at the end of the motor.
2. A **drip-proof type: This type** is similar to the screen-protected type. However, it has a cover over the screen to prevent drips of water entering the machine.
3. A **flame-proof type: This** is the type that is often cooled by the conduction of heat through the motor casing.
4. **Pipe-ventilated type, in this type,** air is piped into the motor from a dust-free area. Also, there is internally fitted fan within the motor. This ensures the circulation of air.

Speed control methods of induction motor

An induction motor is practically a constant speed motor. That means, for the entire loading range, change in speed of the motor is quite small. Speed of a DC shunt motor can be varied very easily with good efficiency, but in case of Induction motors, speed reduction is accompanied by a corresponding loss of efficiency and poor power factor. As induction motors are widely being used, their speed control may be required in many applications. **Different speed control methods of induction motor** are explained below.

Induction motor speed control from stator side

1. By changing the applied voltage:

From the torque equation of induction motor,

Mechanical power = $T\omega = 2\pi n T$ because ω = mechanical speed = $2\pi n$

For a DC motor,

$$V = E + I_a R_a \qquad \text{multiplying each term by } I_a$$

$$VI_a = EI_a + I_a I_a R_a$$

$$VI_a = EI_a + {I_a}^2 R_a$$

Now, the mechanical speed is = $EI\_a = 2\pi n T$

That means

$$T = \frac{EI_a}{2\pi n} \text{ Since } V = E + I_a R_a \ , \quad E = V - I_a R_a$$

$$T = \frac{(V - I_a R_a) I_a}{2\pi n}$$

$$T = \frac{VI_a - {I_a}^2 R_a}{2\pi n}$$

Also,

$$T = \frac{m\,x1}{2\pi n_s} X \frac{S \cdot \frac{{N_2}^2}{{N_1}^2} X {E_1}^2}{{R_2}^2 + (SX_2)^2} \frac{R_2}{.}$$

Rotor resistance R_2 is constant. If slip s is small then $(sX_2)^2$ is so small that it can be neglected. Therefore, $T \propto sE_2^2$

Where

E_2 is rotor induced e.m.f and $E_2 \propto V$

Thus, $T \propto sV^2$,

This means that if the supply voltage is decreased, the developed torque will also decrease. Hence, for providing the same load torque, the slip increases with decrease in voltage, and consequently, the speed decreases. This method is the easiest and cheapest, still rarely used, because

1. Large change in supply voltage is required for relatively small change in speed.
2. Large change in supply voltage will result in a large change in flux density, hence, this will disturb the magnetic conditions of the motor.

2. By changing the applied frequency

Synchronous speed of the rotating magnetic field of an induction motor is given by,

$$N_s = \frac{120 f}{P} \quad \text{(RPM)}$$

Where,

f = frequency of the supply

P = number of stator poles

Hence, the synchronous speed changes with change in supply frequency.

Actual speed of an induction motor is given as

$$\mathbf{N = N_s\,(1 - s)}$$

This method is not widely used. But it may be used where the induction motor is supplied by a dedicated generator (so that frequency can be easily varied by changing the speed of prime mover)

Also, at lower frequency, the motor current may become too high due to decreased reactance. And if the frequency is increased beyond the rated value, the maximum torque developed falls while the speed rises.

3. Constant V/F (variable voltage, variable frequency method) of induction motor

This is the most popular method for controlling the speed of an induction motor. As in above method, if the supply frequency is reduced keeping the rated supply voltage, the air gap flux will tend to saturate. This will cause excessive stator current and distortion of the stator flux wave. Therefore, the stator voltage should also be reduced in proportional to the frequency so as to maintain the air-gap flux constant. The magnitude of the stator flux is proportional to the ratio of the stator voltage and the frequency. Hence, if the ratio of voltage to frequency is kept constant, the flux remains constant. Also, by keeping V/F constant, the developed torque remains approximately constant. This method gives higher run-time efficiency. Therefore, majority of AC speed drives employ constant V/F method (or variable voltage, variable frequency method) for the speed control. Along with wide range of speed control, this method also offers 'soft start' capability.

4. Changing the number of stator poles

From the above equation of synchronous speed, it can be seen that synchronous speed (and hence, running speed) can be changed by changing the number of stator poles. This method is generally used for squirrel cage induction motors, as squirrel cage rotor adapts itself for any number of stator poles. Change in stator poles is achieved by two or more independent stator windings wound for different number of poles in same slots.

For example, a stator is wound with two 3phase windings, one for 4 poles and other for 6 poles. For supply frequency of 50Hz,

i) Synchronous speed when 4 pole winding is connected, Ns = $\frac{120*50}{4}$

= 1500 rev/min

ii) Synchronous speed when 6 pole winding is connected, Ns = $\frac{120*50}{6}$

= 1000 rev/min

SPEED CONTROL FROM ROTOR SIDE:

1. Rotor rheostat control

This method is similar to that of armature rheostat control of DC shunt motor. But this method is only applicable to slip ring motors, as addition of external resistance in the rotor of squirrel cage motors is not possible.

2. Cascade operation

In this method of speed control, two motors are used. Both are mounted on a same shaft so that both run at same speed. One motor is fed from a 3phase supply and the other motor is fed from the induced e.m.f in first motor via slip-rings. The arrangement is as shown in following figure.

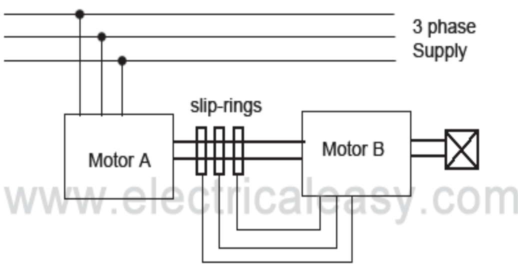


Figure 3.20: speed control by cascade method Reference: www.electricaleasy.com
Motor A is called the main motor and motor B is called the auxiliary motor.

Let,

N_{sync1}= synchronous speed of magnetic field of motor A or frequency of motor A

N_{sync2}= synchronous speed of magnetic field of motor B or frequency of motor B

P_1 = Number of poles of motor A

P_2 = Number of poles of motor B

N_m = Mechanical shaft speed of the rotor set and it is the same for both motors,

f= frequency of the supply in hertz

Now,

The *slip speed* of motor A = $n_{slip} = N_{sync1} - N_m$

Slip of a motor = s = $\frac{n_{slip}}{N_{sync1}} = \frac{Nsync1 - Nm}{N_{sync1}}$

The slip of motor A = s_1= $\frac{n_{slip}}{N_{sync1}} = \frac{Nsync1 - Nm}{N_{sync1}}$

The frequency of the rotor induced e.m.f in motor A = f_1

$$f_1 = sf$$

Likewise the induced rotor voltage at locked-rotor condition is E_{ro}

The induced rotor voltage at any slip is E_r

$$E_r = sE_{ro}$$

Also, $f_r = sf_e$

$$f_r = \frac{(Nsync1 - Nm) \text{ x } f_e}{N_{sync1}}$$

$N_s = \frac{120 f_e}{\text{P}}$ since $f_r = sf_e$

$$N_s = \frac{120 f_r}{\text{Px s}}$$

3. By injecting EMF in rotor circuit

In this method, speed of an induction motor is controlled by injecting a voltage in rotor circuit. It is necessary that voltage (emf) being injected must have same frequency as of the slip frequency. However, there is no restriction to the phase of injected emf. If we inject emf which is in opposite phase with the rotor induced emf, rotor resistance will be increased. If we inject emf which is in phase with the rotor induced emf, rotor resistance will decrease. Thus, by changing the phase of injected emf, speed can be controlled. The main advantage of this method is a wide range of speed control (above normal as well as below normal) can be achieved. The emf can be injected by various methods such as Kramer system, Scherbius system etc

CHAPTER FOUR

INDUCTION MOTORS

Three-phase induction motors

Introduction

In D.C motors, conductors on a rotating armature pass through a stationary magnetic field. In a three-phase induction motor, the magnetic field rotates and this has the advantage that no external electrical connections to the rotor need be made. Its name is derived from the fact that the current in the rotor is **induced** by the magnetic field instead of being supplied through electrical connections to the supply. The result is a motor which is:

1. cheap and robust
2. explosion proof, due to the absence of a commutator or slip-rings and brushes with their associated sparking
3. requires little or no skilled maintenance, and
4. has self-starting properties when switched to supply with no additional expenditure on auxiliary equipment.

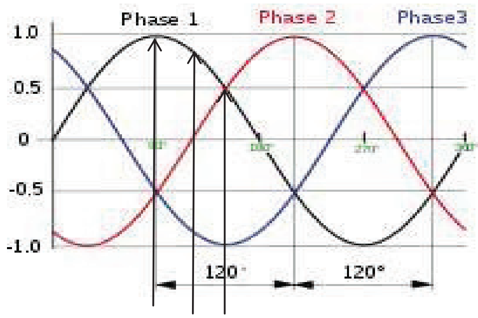
The principal disadvantage of a three-phase induction motor is that its speed cannot be readily adjusted.

PRODUCTION OF A ROTATING MAGNETIC FIELD

Whenever a three-phase supply is connected to three-phase windings, the currents flowing in the windings will produce a magnetic field.

1. The magnetic field produced is constant and
2. It will rotates at constant speed. This constant speed is called the synchronous speed.

Current, I $\quad I_A I_B I_C$



t_1 t_2 t_3 time, t

Figure 4.1: current flowing in each winding varies with time

Also, if the stator windings are connected to three-phase supply, current flowing in each winding will change with time.

1. If the value of current in a winding is positive, we assume that the current flows from start to finish of the winding. That means, if it is the blue phase, current flows from B_S to B_F, i.e. away from the viewer in B_S and towards the viewer in B_F.
2. But if the value of current is negative, the assumption is made that it flows from finish to start, i.e. towards the viewer in an 'S' winding and away from the viewer in an 'F' winding
3. At time, say,t_1, the current flowing in the phase 1 is a maximum positive value. At the same time,t_1, the currentsflowing in the phase 2 and phase 3 are both 0.5 times the maximum value and are negative.

4. The current distribution in the stator windings is therefore as shown below

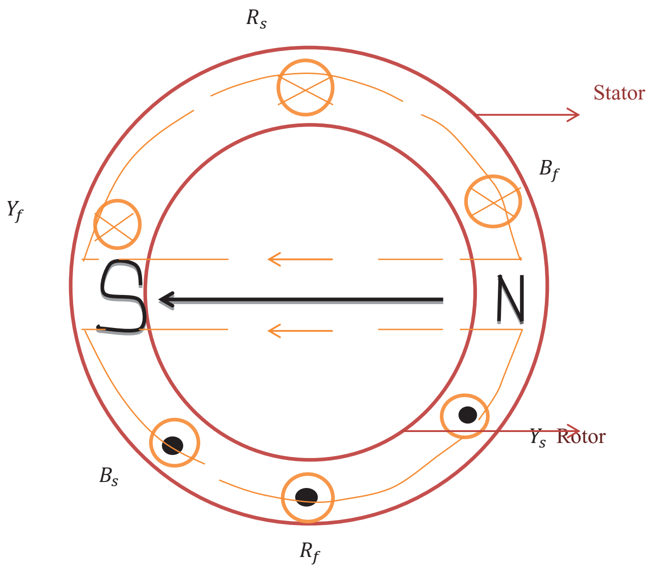


Figure 4.2: Showing current distribution in the stator windings

Current flows away from the viewer, as shown as ⊗ in R_s, Y_f, B_f since it is positive, but towards the viewer as shown as ⊙ in Y_s, B_s , R_f since these are negative. The resulting magnetic field is as shown, due to the 'solenoid' action and application of the corkscrew rule.

5. A short time later at time, t_2, the current flowing in the phase 1 has fallen to about 0.87 times its maximum value and is positive, the current in phase2 is zero and the current in phase 3 is about0.87 times its maximum value and is negative. Hence the currents and resultant magnetic field are as shown below:

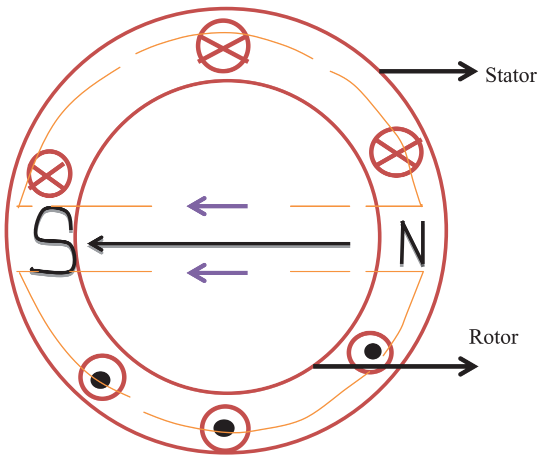


Figure 4.3: **Showing the resultant magnetic field**

6. Now at timet_3, the currents in the phase 1 and phase 3 are 0.5 of their maximum values and the current in phase 2 is a maximum negative value. The currents and resultant magnetic field are as shown below:

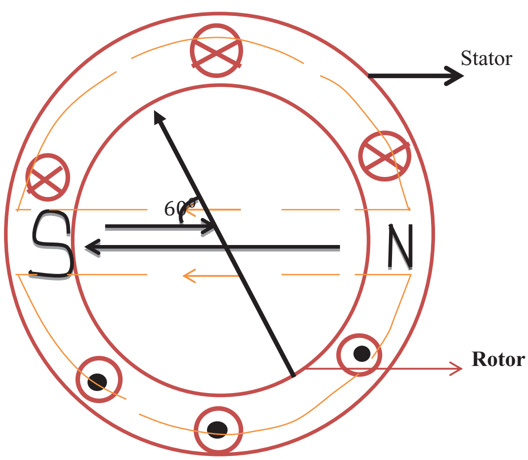


Figure 4.4: Showing the resultant magnetic field at time t_3

By considering the flux values rather than the current values, it can be shown that the rotating magnetic field has a constant value of flux. The three coils are connected in star to a three-phase supply. Let the positive directions of the fluxes produced by currents flowing in the coils, be $Ø_{A,}$ $Ø_{B,}$ $Ø_{C\,,}$ respectively. The directions of $Ø_{A,}$ $Ø_{B,}$ $Ø_C$ do not alter, but their magnitudes are proportional to the currents flowing in the coils at any particular time.
Here, at timet_1, the currents flowing in the coils are:

I_A, a maximum positive value, i.e., the flux is towards point P as shown below

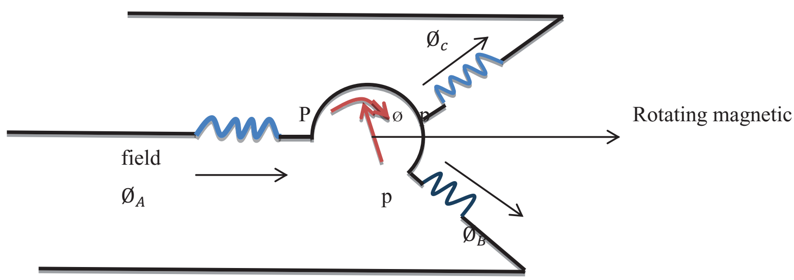
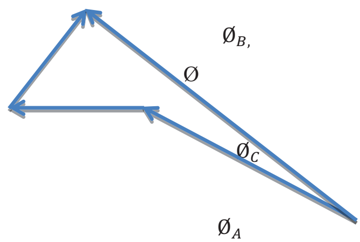


Figure 4.5: Showing the resultant magnetic field

$I_B$and I_C, = half the maximum value and negative, i.e., the flux is away from point P.
These currents give rise to the magnetic fluxes $Ø_{A,}$ $Ø_{B,}$ $Ø_{C\,,}$ whose magnitudes and directions are as shown in the figure below.
The resultant flux is the phasor sum of $Ø_{A,}$ $Ø_{B,}$ $Ø_{C\,,}$ shown as Ø in the figure below:



At timet_1, the currents flowing are:

I_A *which* is a maximum positive value,

I_B = 0.5 x maximum negative value and

I_C= 0.5 x maximum negative value

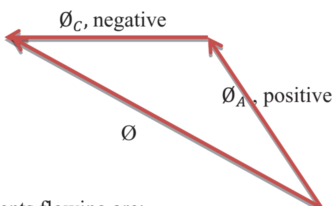
The magnetic fluxes and the resultant magnetic flux are as shown in the figure above:

At timet_2, the currents flowing are:

I_A = 0.866 x maximum positive value, I_B = 0 and

I_C= 0.866 x maximum negative value

The magnetic fluxes and the resultant magnetic flux are as shown in the figure below:



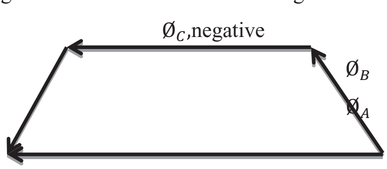
At timet_3, the currents flowing are:

I_A = 0.5 x maximum positive value,

I_B = 0.5 x maximum positive value, and

$I_C$is maximum negative value

The magnetic fluxes and the resultant magnetic flux are as shown in the figure below:



Inspection of the three figures above shows that the magnitude of the resultant magnetic flux, Ø, in each case is constant and it is 1.5 x the maximum value the maximum value of $Ø_{A,}$ $Ø_{B,}$ $Ø_{C\,,}$ but that its direction is changing. The process of determining the resultant flux may be repeated for all values of time and shows that the magnitude of the resultant flux is constant for all values of time and also that it rotates at constant speed, making one revolution for each cycle of the supply voltage.

Synchronous speed

The rotating magnetic field produced by three phase windings will produce the same magnetic field by rotating a permanent magnet's north and south pole at synchronous speed, Therefore, it is called a 2-pole system and an induction motor using three phase windings will be referred to as a 2-pole induction motor.

If we use a six windings machine, it will be observed that one cycle of the supply current supplied to the stator windings will make the magnetic field to rotate only half a revolution. The current distribution in the stator windings are shown in the figure below:

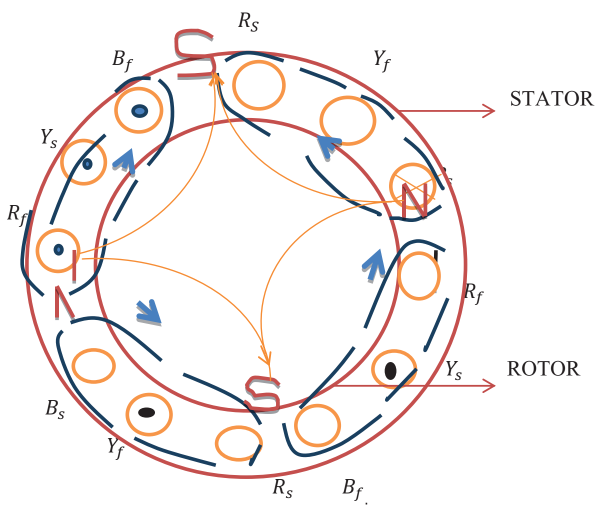
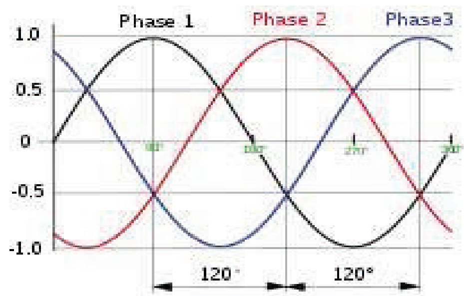


Figure 4.6: current distribution in the stator windings

The current distribution in the stator windings are shown in the figure below

Current I



$t_1$time, t, sec

Figure 4.7: Currents waveform in the stator windings

It will be seen that if we have six windings on the stator, the magnetic flux produced is the same as that produced by rotating two permanent magnet north poles and two permanent magnet south poles at synchronous speed. This is therefore called a 4-pole system. Hence,an induction motor using six phase windings is called a 4-pole induction motor. If we increase the number of phase windings, the number of poles can be increased.

In general, if f is the frequency of the currents in the stator windings and the stator is wound to P poles, the speed of revolution of the rotating magnetic field which is the synchronous speed(n_s) in rev/sec is given by:

$$n_s = \frac{f}{p}\textbf{Rev/s}$$

You will remember that

A six winding electrical machine will produce the same magnetic field as a 4 poles electrical machine. Therefore, 4 pole electrical; machine is the same as six winding electrical machine.

In practice, when a three phase set of current is applied to the stator windings of 4 poles or six windings AC or DC machine, poles North and South in the machine, the pole will move around(rotate around) half the stator surface or half revolution i.e 180^o in one electrical surface.

Since the mechanical motion in 4 poles electrical machine is 180^o for every electrical cycle of 360^o.

Then in 4 poles electrical machine,

The electrical angle $\boldsymbol{\theta}_e$ of 360 ^{o} is = 2 x mechanical angle $\boldsymbol{\theta}_m$ of 360^o

orelectrical angle $\boldsymbol{\theta}_e$ is $= \frac{4}{2}$ x mechanical angle $\boldsymbol{\theta}_m$

orelectrical angle $\boldsymbol{\theta}_e$ is $= \frac{P}{2}$ x mechanical angle $\boldsymbol{\theta}_m$

Finally,

$\boldsymbol{\theta}_e = 2\,\boldsymbol{\theta}_m$ for 4 poles

Since frequency means the number of revolutions completed in one second, the electrical frequency is also double the mechanical frequency for 4 poles

$f_e = 2\,f_m$ for 4 poles

Since angular velocity $w = 2\pi f$

The electrical angular velocity is also double the mechanical angular velocity for 4 poles

$w_e = 2\,w_m$ for 4 poles

TAKE NOTE THAT:

7 If there are P numbers of magnetic poles on the stator of an electromagnetic machine, then there will be $\frac{P}{2}$ repetition of the windings.

8 $\theta_e = \frac{P}{2} \times \theta_m$

9 $f_e = \frac{P}{2} f_m$

10 $w_e = \frac{P}{2} w_m$

11 Mechanical frequency = f_m in revolution per second

Mechanical speed in revolution per minute = N_m

Then,

$$f_m = \frac{N_m}{60}$$

12 Comparing number 3 and 5 above

$$f_e = \frac{P}{2} f_m \quad , \qquad f_m = \frac{N_m}{60}$$

$$f_e = \frac{P}{2} \times \frac{N_m}{60}$$

$$f_e = \frac{P\,N_m}{120}$$

$$N_m = \frac{120 f_e}{P} \qquad \text{P= number of poles}$$

13 Also, $N_m = \frac{60 f_e}{p}$

p = number of pairs of pole i.e $p = \frac{P}{2}$

14 Let the mechanical speed of the machine in revolution per second be= n_m

Then,

15 $n_m = \frac{N_m}{60}$

Since $N_m = \frac{60 f_e}{p}$

Then, $n_m = \frac{60 f_e}{p x 60}$

Hence,

16 $n_m = \frac{f_e}{p}$

Example 1

A three-phase two-pole induction motor is connected to a 60 Hz supply. Determine the synchronous speed of the motor in rev/min.

Solution:

Since,

$$n_m = \frac{f_e}{p}$$

n_m =Synchronous speed in rev/sec

N_m = synchronous speed in rev/min

$f_e = electrical\ frequency\ of\ the\ power\ supply\ to\ the\ motor$

$p = number\ of\ pairs\ of\ poles = \frac{P}{2}$

P =Number of poles

Hence,

$$n_m = \frac{60}{1}$$

n_m= 60 rev/sec or 3600 rev/min

Example2

A stator winding supplied from a three-phase 50 Hz system is required to produce a magnetic flux rotating at 1500 rev/min. Determine the number of poles.

Since,

N_m = 15f00 rev/min

$f_e =$

$$n_m = \frac{f_e}{p}$$

We know that $n_m = \frac{N_m}{60}$

Then, $n_m = \frac{1500}{60} n_m$ = 25 rev/sec

$$n_m = \frac{f_e}{p}$$

25 $= \frac{50}{p}$, then $p = 2$

The number of poles = 2 p = 4

Example3

A three-phase 2-pole motor is to have a synchronous speed of 3600 rev/min. Calculate the frequency of the supply voltage.

$$n_m = \frac{f_e}{p}$$

$$\frac{3600}{60} = \frac{f_e}{1}$$

Therefore the frequency is $= \frac{3600}{60}$

Frequency is = 60 Hz

CONSTRUCTION OF A THREE-PHASE INDUCTION MOTOR

The stationary part of a three-phase induction motor is the stator or the yoke. It is wound to give a 2-pole, 4-pole, 6-pole,8-pole . . . . . . rotating magnetic field, depending on the rotor speed required. The rotor, corresponding to the armature of a D.C machine, is built up of laminated iron, to reduce eddy currents.

Types of induction motors

1. **squirrel-cage rotor induction motor**
2. **wound rotor induction motor**

The type most widely used is the **squirrel-cage rotor**, in such motor, copper or aluminum bars are placed in slots cut in the laminated iron, the ends of the bars being welded or brazed into a heavy conducting ring as shown below:

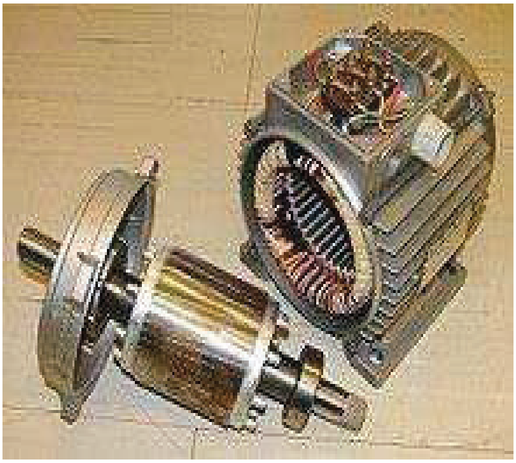


Figure 4.8: Squirrel-cage rotor inductor, Reference: www.electricaleasy.com

The constructional feature of an electric motor can be represented as shown below

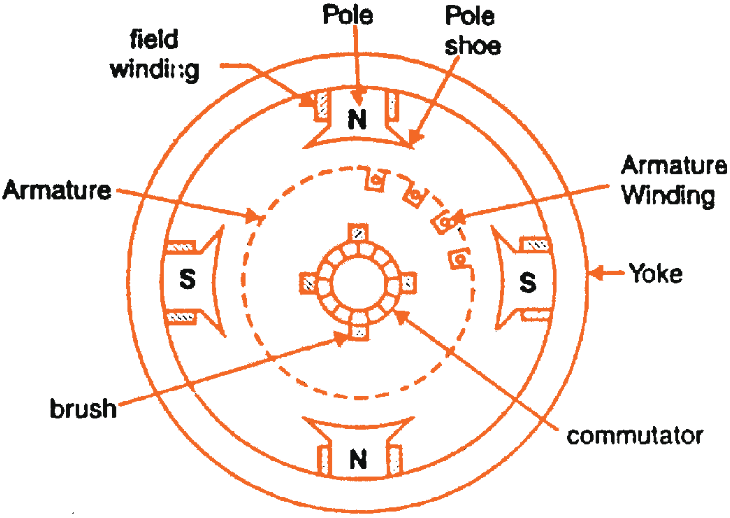


Figure 4.9: shows the Constructional feature of a DC Machine

The conductors are placed in slots in the laminated iron rotor core. If the slots are skewed, better starting and quieter running is achieved. These types of rotor do not have external connections. That means slip rings and brushes are not needed.

Advantages of squirrel-cage motor

1. it is cheap,
2. it is reliable and
3. it has high efficiency.

WOUND ROTOR

This is another type of rotor is the **wound rotor.** With this type, there are phase windings in slots, similar to those in the stator. The windings maybe connected in star or delta and the connections made to three slip rings. The slip rings are used to add external resistance to the rotor circuit, particularly for starting.

The principle of operation is the same for both the squirrel cage and the wound rotor machines.

Principle of operation of a three-phase induction motor

1. In induction and synchronous motors, AC power supplied to the motor's stator winding will creates a magnetic field that rotates in time with the AC waveform.
2. The rotor of a synchronous motor rotates at the same rate as the stator field. But the rotor of induction motor will rotate at a slower speed than the speed of the stator field.
3. The magnetic field of the stator of an induction motor $thai\ is\ B_S$ is therefore changing relative to the rotor. This induces an opposing current in the rotor of the induction motor.
4. The rotating magnetic flux induces currents in the windings of the rotor
5. The currents in the rotor windings in turn create magnetic fields B_R in the rotor. This current opposes the current in the stator field.
6. Due to Lenz's Law, the direction of the current induced in the rotor will oppose the direction of the stator magnetic field. The cause of induced current in the rotor windings is the rotating stator magnetic field.
7. In order to oppose the change in rotor-winding currents the rotor will start to rotate in the direction of the rotating stator magnetic field. The rotor will accelerate until the magnitude of induced rotor current and torque balances the applied load.
8. Since rotation at synchronous speed would result in no induced rotor current, induction motor always operates slower than synchronous speed.
9. The difference, or "slip," between actual and synchronous speed varies between 0.5 to 5.0% for standard Design torque curve induction motors.

Slip

The difference between the rotor speed, n_r, and the synchronous speed, n_s is called the**slip speed**

Slip speed = $n_s - n_r$ $\quad$ $in\ rev/sec$

1. The ratio $\frac{n_{s} - n_r}{n_s}$ is called the **fractional slip** or just the **slip**, s,
2. It is usually expressed as a percentage. Hence,

$$\textbf{Slip, } s = \frac{n_{s} - n_r}{n_s} \text{ x } 100\%$$

For small motors, the slip between no load and full load are about 4 to 5%

It is 1.5 to 2% for large motors.

Example 4

The stator of a 3-phase, 8-pole induction motor is connected to a 50 Hz supply. The rotor runs at 720 rev/min at full load. Determine

i. the synchronous speed and

ii. the slip at full load.

Solution

i. $p = number\ of\ pairs\ of\ poles = \frac{P}{2}$

$$p = number\ of\ pairs\ of\ poles = \frac{8}{2} = 4$$

Since electrical frequency of the supply =50 Hz

Hence,

$$n_s = \frac{f_e}{p}$$

$$synchronous\ speed = n_s = \frac{50}{4} = 12.5 \text{ rev/sec}$$

Rotor speed $= n_r = \frac{N\ in\ rev/min}{60}$

Rotor speed= $n_r = \frac{720}{60} = 12$ rev/sec

ii. **slip, *s*** $= \frac{n_{s-}\ n_r}{n_s}$ x 100%

Slip, *s* $= \frac{12.5 - 12}{12.5}$ x 100%

Slip, *s* $= \frac{0.5}{12.5}$ x 100%

Slip, *s* $= \frac{50}{12.5}$ %

Slip, *s* = 4 %

Example 5

A 3-phase, 60 Hz induction motor has 2 poles. If the slip is 3% at a certain load, determine

(a) the synchronous speed,

(b) the speed of the rotor and

(c) the frequency of the induced e.m.f in the rotor.

Solution

a) The synchronous speed $= n_s$, $p = pairs\ of\ pole\ = \frac{number\ of\ pole}{2}$

$$p = pairs\ of\ pole\ = \frac{2}{2} = 1$$

$$n_s = \frac{f_e}{p}$$

$$n_s = \frac{60}{1}$$

n_s= 60 rev/sec

In revolution /minute, the speed of the machine becomes

n_s in rev/sec x 60

= 60 rev/sec x 60

= 3600 rev/minute

b) The speed of the rotor

The slip is 2% at a certain load,

Since,

$$\textbf{Slip, } s = \frac{n_s - n_r}{n_s} \text{ x } 100\%$$

$$3\% = \frac{n_s - n_r}{60} \text{ x } 100\%$$

$$3\% = \frac{n_s - 60}{60} \text{ x } 100\%$$

180% = 60 - n_r x 100%

180 = (60 - n_r)100

$180 = 6000 - 100n_r$

$100n_r = 6000 - 180$

$100n_r = 5820$

$n_r = 58.20$ rev/sec

Hence the speed of the rotor in rev/sec = 58.20rev/se

In rev/min the speed will be = 58.2 × 60 = 34.92 rev/min

(c)The frequency of the e.m.f induced in the rotor

Since the synchronous speed is 60 rev/s and that of the rotor is 58.2rev/s, the rotating magnetic field

cuts the rotor bars at (60−58.2) i.e. 1.8 rev/s.

The difference between the rotor speed, n_r, and the synchronous speed, n_s is called the **slip speed**

Slip speed = $n_s - n_r$ $\quad in\ rev/sec$

That means the rotating magnetic field cuts the rotor bars at a speed equal to this slip speed which is

Slip speed = $n_s - n_r$ $\quad in\ rev/sec$

= 60−58.2

= 1.8 rev/s

Since

$$n_s = \frac{f_e}{p}$$

Thus the frequency of the e.m.f.'s induced in the rotor bars,

$f_e = n_{slip\ speed, speed\ at\ which\ the\ rotatingmag.field\ cut\ the\ rotor\ bar}\ x\ p =$ 1.8 x 1

$f_e =$ 1.8 Hz

Assignment

1) A three-phase induction motor is supplied from a 50 Hz supply and runs at 1400 rev/min when the slip is 2%. Determine the synchronous speed.

2) The frequency of the supply to the stator of an 8- pole induction motor is 50 Hz and the rotor frequency is 2.5 Hz.

Determine (a) the slip

(b) the rotor speed

ROTOR E.M.F. AND FREQUENCY

ROTOR E.M.F.

When an induction motor is stationary, the stator and rotor windings form the equivalent of a transformer as shown below

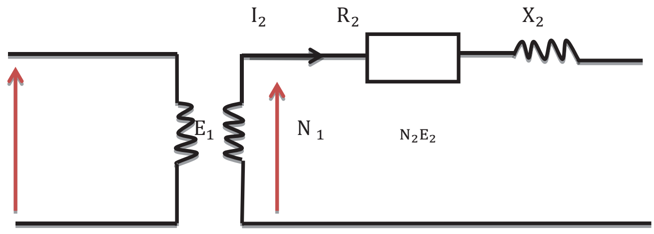


Figure 4.10 : Induction motor in stand-stll stage

The rotor e.m.f, E_2**at standstill** is given by

$$E_2 = \frac{N_2}{N_1} \text{ X } E_1$$

Where: E1 is the supply voltage per phase to the stator.

When an induction motor is running,the induced e.m.f. in the rotor is reduced. This is because the relative movement between conductors and the rotating field is lower. Since the induced e.m.f. is proportional to this movement, hence it must be proportional to the slip, s.

Hence, when running, rotor e.m.f. per phase= E_r

$$= E_r = sE_2 = s \text{ x } \frac{N_2}{N_1} \text{x } E_1$$

ROTOR FREQUENCY

From the fact that the rotor e.m.f. is induced by an alternating flux, the rate at which the flux passes the conductors is the slip speed. Thus the frequency of the e.m.f. induced on the rotor is given by:

$f_r = (n_s - n_r)\, p$ p = number of pairs of pole

If we multiply and divide the above expression by n_s , then

$$f_r = \frac{n_s - n_r}{n_s} \times n_s\, p$$

However

$\frac{n_s - n_r}{n_s}$ is the slip s and, from $n_s = \frac{f}{p}$, n_s Pis the supply frequency f

Hence

$$f_r = sf$$

Example 6

The frequency of the supply to the stator of a 4-pole induction motor is 50 Hz and the rotor frequency is 4 Hz.

Determine (a) the slip

(b) the rotor speed.

(a) Since, slip, $s = \frac{n_s - n_r}{n_s} \times 100\%$ and it has been proved that $f_r = s f$

f_r = rotor frequency = 4Hz

s = slip

f = electrical frequency = 50 Hz

Hence the slip is equal to $s = \frac{f_r}{f}$

$$s = \frac{4}{50}$$

S = 0.08

In percentage S = 0.08 X 100 = 8%

b) Since

$$n_s = \frac{f_e}{p}$$

p = pairs of pole = $\frac{4}{2}$= 2, f_e = electrical frequency = 50Hz

Therefore,

$$n_s = \frac{50}{2} = 12.5$$

$$\text{Slip, } s = \frac{n_{s-}\, n_r}{n_s}$$

$$0.08 = \frac{12.5 - n_r.}{12.5}$$

$$1 = 12.5 - n_r$$

$$n_r = 12.5 - 1$$

$$n_r = 11.5 \text{ rev/second}$$

In rev/minute $n_r = 11.5$ rev/second x 60

$n_r = 11.5$ rev/second x 60 = 690 rev/minute

Rotor impedance and current

From the circuit diagram below

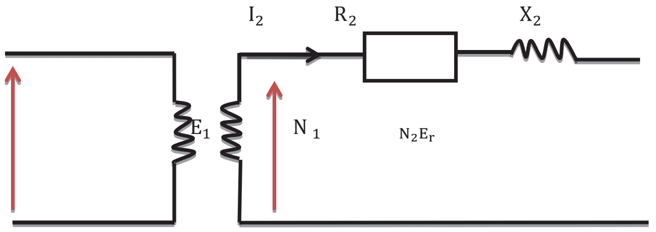


Figure 4.11: Rotor impedance and current

Rotor resistance is R_2. It will not be affected by change in frequency or slip. Therefore, it will always remain constant.

The rotor e.m.f **at standstill** is given byE_2

$$E_2 = \frac{N_2}{N_1} \text{ X } E_1$$

Where: E1 is the supply voltage per phase to the stator.

When the motor is running, the relative movement between the (now moving) conductors and the rotating magnetic field will be reduced. The rotor rotates at a speed equal to n_r *and* n_r *is* a little lower than the synchronous speed that is *equal to* n_s . The difference

$n_s - n_r$ *is the slip speed.*

$$This\ slip\ speed, n_s - n_r\ is\ the\ speed\ of\ the\ e.m.f\ induced\ on\ the\ rotor$$

1. Since $n_s = \frac{f}{p}$ where f = frequency of the supply e.m.fE1
2. $speed\ of\ the\ e.m.f\ induced\ on\ the\ rotor = n_s - n_r$
3. $Hence, n_s - n_r = \frac{f_r}{p}$
4. $f_r = n_s - n_r\ x\ p$
5. Since, $n_s = \frac{f}{p}, p = \frac{f}{n_s}$
6. f_r that was $= n_s - n_r\ x\ p$ will now be

$$f_r = n_s - n_r \; x \; \frac{f}{n_s}$$

$$f_r = \frac{n_s - n_r}{n_s} \; x \; f \qquad \text{since, Slip, s} = \frac{n_s - n_r}{n_s}$$

7. $f_r = s \; x \; f$ this is the rotor frequency at running stage
8. **Rotor resistance is** R_2 will not be affected by change in frequency or slip. Therefore, it will always remain constant. But the rotor impedance will be affected by the slip s
9. Hence, rotor reactance at running stage = X_r and it is now = sX_2
10. The operating current will also change from I_2 to I_r rotor current

During the running stage

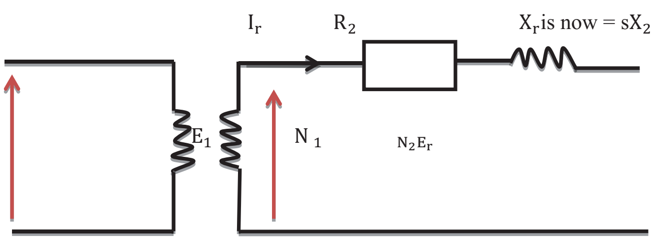


Figure 4.12a -c: Three phase induction motor during running stage

1. As we have said earlier, rotor resistance is r_2 will not be affected by change in frequency or slip.

   Therefore, it will always remain constant. But the rotor impedance will be affected by the slip s

2. Hence, rotor reactance at running stage = X_r and it is now = sX_2

3. E.M.F induced on the rotor or rotor e.m.f = E_r and $E_r = s \; E_2$

Since, $E_2 = \frac{N_2}{N_1} \; X \; E_1$ $\qquad E_r = s \; \frac{N_2}{N_1} \; X \; E_1$

4. $X_2 = 2\pi fl$

5. Since rotor reactance at running stage = X_r and it is now = sX_2

   $X_r = sX_2$

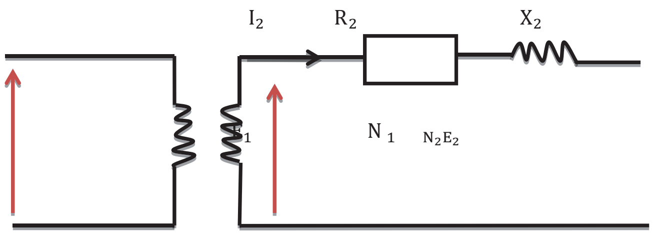
   $X_r = s(2\pi fl)$

6. Rotor impedance at stand still as shown below is

$$Z_2 = \sqrt{R_2{}^2 + X_2{}^2}$$

Also, at stand still, the rotor speed = 0. This is because the rotor is not moving.

Hence, slip s = $\frac{n_s - n_r}{n_s}$ *will become* s = $\frac{n_s - 0}{n_s} = \frac{n_s}{n_s} = 1$



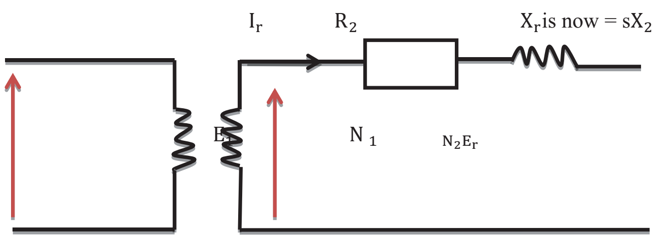
Also,

$$I_2 = \frac{E_2}{Z_2}$$

Since, $E_2 = \frac{N_2}{N_1} X E_1$ and $Z_2 = \sqrt{R_2{}^2 + X_2{}^2}$

$$I_2 = \frac{\left(\frac{N_2}{N_1}\right) E_1}{\sqrt{R_2{}^2 + X_2{}^2}}$$

7. During the running stage $Z_r = \sqrt{R_2{}^2 + X_r{}^2}$



8. Here, the Rotor impedance at running stage is

$$Z_r = \sqrt{R_2{}^2 + X_r{}^2}$$

Since, $X_r = sX_2$

$$Z_r = \sqrt{R_2{}^2 + (sX_2)^2}$$

Also,

$$I_r = \frac{E_r}{Z_r}$$

9. Since, $E_r = s\ E_2$ and $E_2 = \frac{N_2}{N_1} X E_1$

$$E_r = s\ \frac{N_2}{N_1}\ X\ E_1$$

$$Z_r = \sqrt{R_2{}^2 + (sX_2)^2}$$

Then, $I_r = \frac{E_r}{Z_r}$ will become

$$I_r = \frac{s\frac{N_2}{N_1}\ X\ E_1}{\sqrt{R_2{}^2 + (sX_2)^2}}$$

THE LOSSES IN AN INDUCTION MOTOR

The figure below shows

1. The input power into the stator of an electric motor = P_1,
2. The stator power losses
3. The input power to the rotor = P_2
4. The rotor losses
5. The total power developed by the rotor including friction and windage losses = P_r
6. The friction and windage losses
7. The useful mechanical power from the rotor shaft = P_m

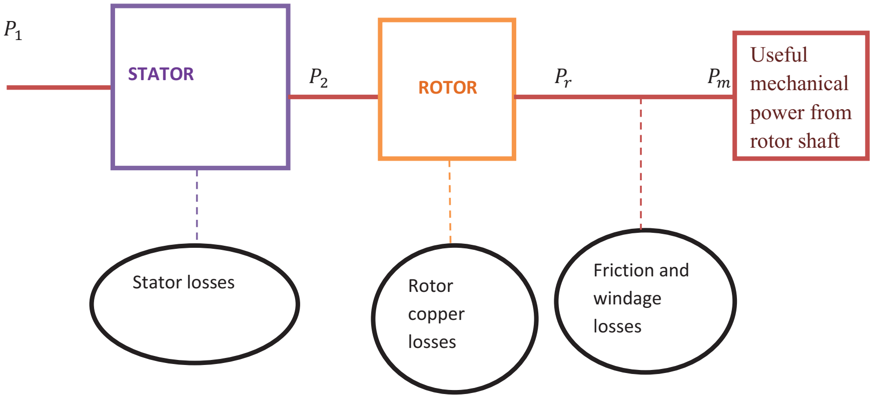


Figure 4.13: Losses in an Induction Motor

It is clear that

1. $P_2 = P_1 -$ stator losses
2. $P_r = P_2 -$ Rotor copper losses
3. $P_m = P_r -$ Friction and windage losses
4. Power output = $P_m - mechanical\ losses$

10. Remember that For a DC motor, if the applied voltage is V. When the DC motor rotates, an e.m.f will be induced in the armature windings. This induced e.m.f is E. From Lenz law, the induced e.m.f E will oppose the motion of the supply voltage that produces it. This e.m.f, E is known as back e.m.f . It is less than the applied voltage as shown in the equation below:

$$E = V - I_a R_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (1)$$

Hence

$$V = E + I_a R_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (2)$$

If we multiply each term in the equation by I_a , then we will obtain

$$VI_a = EI_a + I_a R_a I_a$$

$$VI_a = EI_a + I_a.^2 R_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

Generally in an electric motor,

Electrical power = Mechanical Power + Losses

VI_a= Electrical power

EI_a = Mechanical Power ……………………………..(4)

$I_a.^2 R_a$= Lossesdue to the armature resistance R_a

Also,
The mechanical power is = torque (T) in Newton metres x mechanical speed of the motor (w)

Mechanical power is = T x w……………………….. (5)

Comparing 4 and 5
Mechanical Power = EI_a= T x w since w = $2\pi f$ or w = $2\pi n$

n = armaturespeed in rev/sec

f =Frequency of the motor in rev/sec

Take note that if the loss in the shunt circuit and sum of the iron, friction and windage losses are

neglected

Then, armature speed in rev/sec= the frequency of the rotor in rev/sec

Then, EI_a = T x $2\pi n$ = Mechanical power

$$T = \frac{EI_a}{2\pi n} = \frac{\text{Mechanical Power P}}{2\pi n}$$

Since,

The input power *to the rotor* $from\ manetic\ field\ rotating\ at\ speed\ n_s = P_2$ and

The useful mechanical power *from the rotor shaft* $rotating\ at\ speed\ n_r = P_m$

Then, $T = \frac{EI_a}{2\pi n} = \frac{\text{Mechanical Power P}}{2\pi n}$ will become

$$T = \frac{P_2}{2\pi n_s} \quad \text{or} \quad T = \frac{P_m}{2\pi n_r}$$

That means

$$T = \frac{P_2}{2\pi n_s} = \frac{P_m}{2\pi n_r} \text{ and}$$

$$\frac{P_2}{n_s} = \frac{P_m}{n_r} \text{or}$$

$Subtracting\ each\ quatity\ from\ each\ side\ will\ result\ to:$

$$\frac{P_m}{P_2} = \frac{n_r}{n_s}$$

$$1 - \frac{P_m}{P_2} = 1 - \frac{n_r}{n_s}$$

$$\frac{P_2 - P_m}{P_2} = \frac{n_s - n_r}{n_s}$$

$You\ will\ remeber\ that$ $\frac{n_s - n_r}{n_s}$ is the slip s

$$Hence,\quad slip\ s = \frac{P_2 - P_m}{P_2} = \frac{n_s - n_r}{n_s}$$

Note that the electrical loss or copper loss = $I_r{}^2R_2$

$Note\ that\ P_2 - P_m$ is also the electrical loss or copper loss

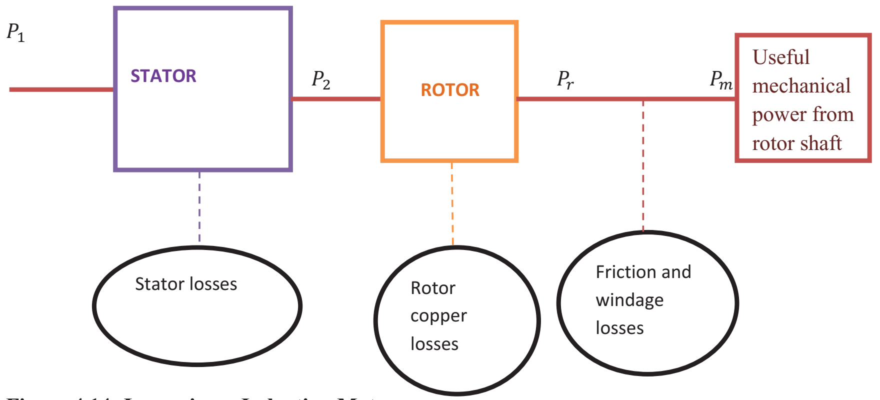


Figure 4.14: Losses in an Induction Motor

11. If $slip\ s = \frac{P_2 - P_m}{P_2} = \frac{n_s - n_r}{n_s}$

and electrical or copper loss=$I_r{}^2 R_2$ or $P_2 - P_m$

Then, $slip\ s = \frac{I_r{}^2 R_2}{P_2} = \frac{rotor\ copper\ loss}{rotor\ input\ power} \frac{I_r{}^2 R_2}{P_2}$

Hence, power input to the rotor = P_2

Then, $P_2 = \frac{I_r{}^2 R_2}{slip\ s}$

12. Electric Motor Efficiency = $\frac{output\ power\ =\ P_m}{input\ power = P_1}$ x 100

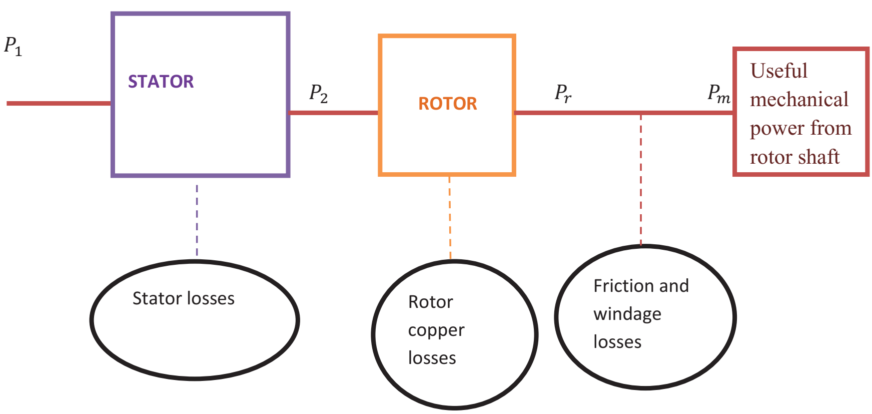
Example 7

The power supplied to a three-phase induction motor is 48kW and the stator losses are 1500W. If the slip is 4%,

Determine s

1. the rotor copper loss
2. the total mechanical power developed by the rotor
3. the output power of the motor if friction and windage losses are 800 W, and
4. the efficiency of the motor, neglecting rotor iron loss.

Solution



1. The input power into the stator of an electric motor = $P_1 = 48$ kW
2. The stator power losses =1500W = 1.5 kW

3. $slip\ s = \frac{P_2 - P_m}{P_2} = \frac{n_s - n_r}{n_s} = 4\% = \frac{4}{100} = 0.04$

4. Electrical loss or rotor copper loss=$I_r{}^2R_2$ or $P_2 - P_m$

5. Then, $slip\ s = \frac{I_r{}^2R_2}{P_2} = \frac{rotor\ copper\ loss}{rotor\ input\ power}\frac{I_r{}^2R_2}{P_2} = 0.04$

6. The input power to the rotor = P_2

7. The rotor losses

8. The total power developed by the rotor including friction and windage losses = P_r

9. The friction and windage losses = 800 W

10. The useful mechanical power from the rotor shaft = P_m

It is clear that

1.
$$P_2 = P_1 - \text{stator losses}$$
$$P_2 = 48 - 1.5$$
$$P_2 = 46.5 \text{ kW}$$

$$Since, \quad slip\ s = \frac{I_r^2 R_2}{P_2} = \frac{rotor\ copper\ loss \quad I_r^2 R_2}{rotor\ input\ power \quad P_2} = 0.04$$

$$0.04 = \frac{I_r^2 R_2}{46.5}$$

$$\text{rotor copper loss} = 0.04 \text{ x } 46.5$$

$$\text{rotor copper loss} = 1.86\ kW = 1860\ W$$

2. The total power developed by the rotor including friction and windage losses = P_r

$P_r = P_2 - \text{Rotor copper losses}$

$$P_r = 46.5 - 1.86$$

$$P_r = 44.64\text{kW}$$

3. output power of the motor $= P_m = P_r - \text{Friction and windage}$

4. losses if friction and windage losses are 800 W = 0.8kW

output power of the motor $= P_m = 44.64 - 0.8$

$P_m = 43.84$ kW

5. Electric Motor Efficiency = $\frac{output\ power =\ P_m}{input\ power = P_1}$

Electric Motor Efficiency = $\frac{43.84\ kW}{48\ \text{kW}}$ x 100

Electric Motor Efficiency = $\frac{4384}{48}$

Electric Motor Efficiency = 91.333 %

TORQUE EQUATION OF AN ELECTRIC MOTOR

1. We have been able to prove that

Torque $T = \frac{EI_a}{2\pi n} = \frac{\text{Mechanical Power P}}{2\pi n}$

$T = \frac{P_2}{2\pi n_s}$ or $T = \frac{P_m}{2\pi n_r}$

2. We also stated that

$slip\ s = \frac{P_2 - P_m}{P_2} = \frac{n_s - n_r}{n_s}$

and electrical or copper loss=$I_r{}^2R_2$ or $P_2 - P_m$

3. Since, $slip\ s = \frac{I_r{}^2R_2}{P_2} = \frac{P_2 - P_m}{P_2} = \frac{rotor\ copper\ loss\ \ I_r{}^2R_2}{rotor\ input\ power\ \ P_2}$

Hence, power input to the rotor = P_2

Then, $P_2 = \frac{I_r{}^2R_2}{slip\ s}$

4. From $T = \frac{P_2}{2\pi n_s} = \frac{1}{2\pi n_s}(P_2)$

$T = \frac{1}{2\pi n_s}(P_2)$ since $P_2 = \frac{I_r{}^2R_2}{slip\ s}$

$T = \frac{1}{2\pi n_s}(\frac{I_r{}^2R_2}{slip\ s})$ since $I_r = \frac{s\frac{N_2}{N_1} X E_1}{\sqrt{R_2{}^2 + (sX_2)^2}}$

$$T = \frac{1}{2\pi n_s} \frac{s^2 \cdot \frac{N_2{}^2}{N_1{}^2} XE_1{}^2}{R_2{}^2 + (SX_2)^2} \frac{R_2}{s}$$

$$T = \frac{1}{2\pi n_s} \frac{s \cdot \frac{N_2{}^2}{N_1{}^2} \cdot XE_1{}^2}{R_2{}^2 + (SX_2)^2} \frac{R_2}{.}$$

If there are m phases, then multiply the above expression by m

In that case

$$T = \frac{m\,x1}{2\pi n_s} \cdot \frac{s \cdot \frac{N_2{}^2}{N_1{}^2} \cdot XE_1{}^2}{R_2{}^2 + (SX_2)^2} \frac{R_2}{.}$$

$$T = \frac{m\,x1}{2\pi n_s} \cdot \frac{s \cdot \frac{N_2{}^2}{N_1{}^2} \cdot XE_1{}^2}{R_2{}^2 + (SX_2)^2} \frac{R_2}{.}$$

$\frac{\frac{mN_2{}^2}{N_1{}^2}}{2\pi n_s}$ is constant , let this be equal to **k**

Then,

$$T = \frac{Ks \cdot XE_1{}^2}{R_2{}^2 + (SX_2)^2} \frac{R_2}{.}$$

Hence,

$$T \,\alpha\, \frac{s \cdot XE_1{}^2}{R_2{}^2 + (SX_2)^2} \frac{R_2}{.}$$

MAXIMUM TORQUE

Note that there will be maximum torque when ${R_2}^2 = (SX_2)^2$

Now, ifmaximum torque occurs when ${R_2}^2 = (SX_2)^2$

Then, maximum torque occurs when $\boldsymbol{R_2}$= s$\boldsymbol{X_2}$ **since** $\boldsymbol{X_r}$ **= s** $\boldsymbol{X_2}$

maximum torque occurs when $\boldsymbol{R_2}$= s$\boldsymbol{X_2}$ = $\boldsymbol{X_r}$

maximum torque occurs when **s** = $\frac{\boldsymbol{R_2}}{\boldsymbol{X_2}}$

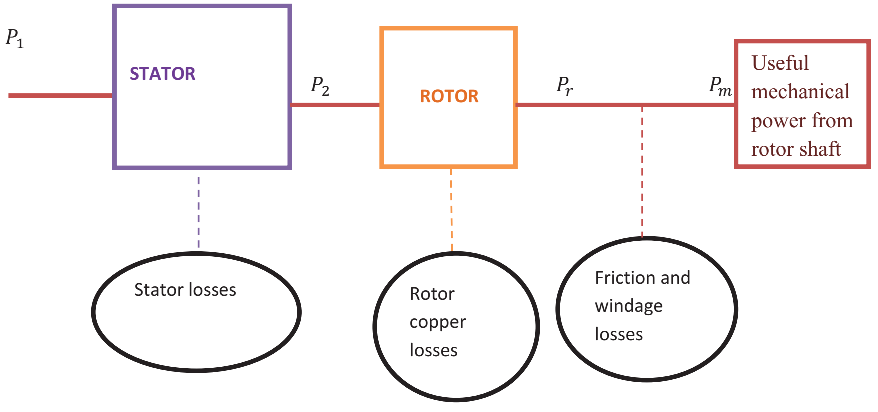
Example 8

A 415V, three-phase, 60Hz, 4-pole, star-connected induction motor runs at 28 rev/s on full load. The rotor resistance and reactance per phase are 0.32 Ω and 3.6 Ω respectively, and the effective rotor-stator turns ratio is 0.8:1

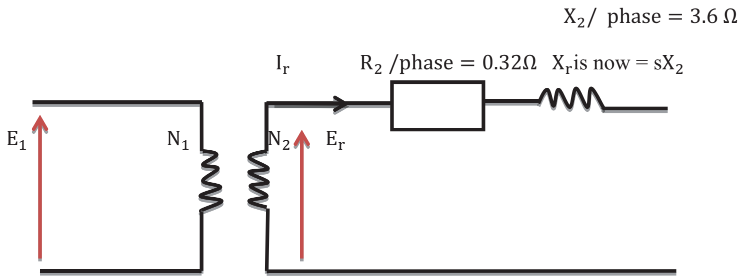
Calculate

1. the synchronous speed
2. the slip
3. the full load torque
4. the power output if mechanical losses amount to 810W
5. the maximum torque
6. the speed at which maximum torque occurs, and
7. the starting torque

Solution



Also, the circuit diagram will be in this form



$$Z_r = \sqrt{{R_2}^2 + {X_r}^2}$$

1. f = 60Hz
2. number of pole = **P** = 4 , then number of pairs of pole = $p = \frac{\mathbf{P}}{2} = \frac{4}{2} = 2$
3. rotor speed = n_r = 28 rev/s or 28 Hz
4. **Since $n_s = \frac{f}{p}$, then $n_s = \frac{60}{2}$ synchronous speed = n_s = 30 rev/sec or 1800 rev/min**
5. $V_L = 415\ V$
6. For star connection,

Phase voltage = $V_P = \frac{V_L}{\sqrt{3}}$ in the circuit diagram, $V_P = E_1$

Phase voltage = $V_P = \frac{415}{\sqrt{3}}$

$V_P = E_1 = 239.6$ V

7. Number of phase = m = 3
8. Turns ratio = $\frac{N_2}{N_1} = \frac{0.8}{1}$

$R_2 = 0.32\Omega$, and $X_2/$ phase = 3.6 Ω

9. X_r is = sX_2

$X_r = sX_2$

$$slip\ s = \frac{P_2 - P_m}{P_2} = \frac{n_s - n_r}{n_s}$$

$$slip\ s = \frac{n_s - n_r}{n_s}$$

$$slip\ s = \frac{30-28}{30} = \frac{2}{30} = 0.0667$$

Percentage slip = 0.0667 x100 = 6.67 %

10. $X_r = sX_2$

$X_r = 0.0667\ (3.6\ )$

$X_r = 0.0667\ (3.6\ ) = 0.24\ \Omega = sX_2$

11. Full load torque = T

$$\text{And}\quad T = \frac{m\ x1}{2\pi n_s} X \frac{s \cdot \frac{N_2^{\ 2}}{N_1^{\ 2}} X E_1^{\ 2}}{R_2^{\ 2} + (SX_2)^2} \frac{R_2}{.}$$

$$T = \frac{3\ x1}{2\ x\ 3.142\ x\ 30} X \frac{0.0667\ x \frac{0.8}{1} x \frac{0.8}{1} X\ 239.6^2 x}{0.32^2 + (0.24)^2} \frac{0.32}{.}$$

NB: X = multiplication

$$T = \frac{2352.62}{2x\ 3.142x30x0.16}$$

$$T = \frac{2352.62}{30.1632} = 78\ \text{Newton metre}$$

12. power output = if mechanical losses amount to 810W = 0.81kW

Since, $T = \frac{P_m}{2\pi n_r}$

$$78 = \frac{P_m}{2x\ 3.142\ x\ 28}$$

P_m = 78 x2x 3.142 x 28

P_m = 13,736.6 Watts

Power output = P_m – $mechanical\ losses$

Power output = 13.7366 – 0.81

Power output = 13,736.6– 810 = 12,926.6 Watts

13. Maximum torque

Since,

$$T\ \alpha\ \frac{s \cdot XE_1^2}{R_2^2 + (SX_2)^2} \cdot \frac{R_2}{}$$

Note that there will be maximum torque when $R_2^2 = (SX_2)^2$

Now, if maximum torque occurs when $R_2^2 = (SX_2)^2$

Then, maximum torque occurs when **R_2= sX_2 since X_r = s X_2 i.e. when**

R_2 = 0.32 = s X_2

maximum torque occurs when **R_2= sX_2 = X_r**

maximum torque occurs when **s = $\frac{R_2}{X_2}$ i.e. when s = $\frac{0.32}{3.6}$ = 0.089**

Hence, Maximum torque $$\mathbf{T = \frac{m\,x1}{2\pi n_s} \cdot \frac{s \cdot \frac{N_2^2}{N_1^2} \cdot E_1^2}{R_2^2 + (SX_2)^2} \cdot \frac{R_2}{}}$$

$$T = \frac{3 \text{ x} 1}{2 \text{ x } 3.142 \text{ x } 30} X \frac{0.089 \cdot \text{x} \frac{0.8}{1} \text{x} \frac{0.8}{1} \text{X } 239.6^2 \text{x}}{0.32^2 + (0.32)^2} \frac{0.32}{.}$$ *NB: X = multiplication*

$$T = \frac{3139.17}{2x\ 3.142x30x0.2048}$$

$$T = \frac{3139.17}{38.61} = 81.3 \text{ Newton metre}$$

14. Speed at which maximum torque occurs

Since, $slip\ s = \frac{n_s - n_r}{n_s}$

maximum torque occurs when $s = \frac{R_2}{X_2}$ **i.e. when** $s = \frac{0.32}{3.6}$ **= 0.089 or when s =8.9% when expressed in percentage**

Since $n_s = \frac{f}{p}$, **then** $n_s = \frac{60}{2}$, **synchronous speed =** n_s **= 30 rev/sec or 1800 rev/min**

$$slip\ s = \frac{n_s - n_r}{n_s}$$

$$0.089 = \frac{30 - n_r}{30}$$

$2.67 = 30 - n_r$

Speed at which maximum torque occurs (n_r) = 27.33 rev/sec

15. the starting torque
Before you start the motor, the rotor is at stand still. Hence, $n_r = 0$

Since, $slip\ s = \frac{n_s - n_r}{n_s}$

$$slip\ s = \frac{n_s - 0}{n_s}$$

$$slip\ s = \frac{n_s}{n_s} = \frac{30}{30} = 1$$

At stand still, we have seen that s=1, $Z_r = \sqrt{R_2^{\ 2} + X_r^{\ 2}}$

If there are m phases , then multiply the above expression by m

In that case

$$\mathrm{T} = \frac{m\ x1}{2\pi n_s} X \frac{s \cdot \frac{N_2^{\ 2}}{N_1^{\ 2}} \cdot XE_1^{\ 2}}{R_2^{\ 2} + (X_2)^2} \frac{R_2}{.} \quad NB{:}X = multiplication$$

$$\mathrm{T} = \frac{3\ x1}{2\ x\ 3.142\ x\ 30} X \frac{1 \cdot x \frac{0.8}{1} x \frac{0.8}{1} X\ 239.6^2 x}{0.32^2 + (3.6)^2} \frac{0.32}{.}$$

$$\mathrm{T} = \frac{35271.57}{2\ x\ 3.142\ x\ 30\ X\ 13.062}$$

$$\mathrm{T} = \frac{35271.57}{2462.52} = 14.32 \text{ Newton metre}$$

Note that

1. **Full load torque = 78 Newton metre and Full load speed = 28 rev/sec**
2. **Whereas** the starting torque = **14.32 Newton metre. Here, the speed is zero because the rotor is at stand still**
3. Maximum torque **= 81.3 Newton metre,** Speed at which maximum torque occurs = 27.33 rev/sec
4. Synchronous speed is 30 rev/sec

We can now draw the torque speed characteristic as shown below:

	Starting or stand still or rotor lock stage	Full load	Maximum torque
Torque	14.32 Nm	78 Nm	81.3 Nm
Speed	0	28 rev/sec	27.33 rev/sec

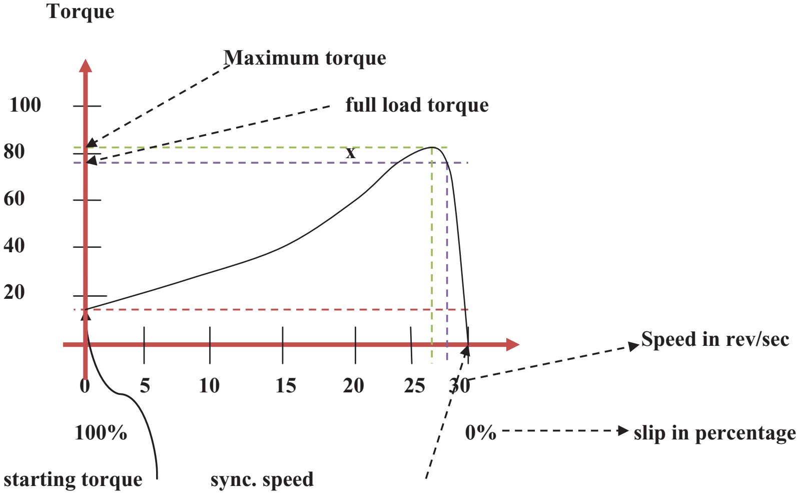


Figure 4.15: Torque speed characteristic

It is clear from this torque speed characteristic that at synchronous speed, torque will be = 0, slip will also be zero

Also, maximum torque occurs at high speed of 81.3 Nm

The curve cuts the full load torque line at point x

If a maximum torque is required at starting, then a high resistance rotor is needed. Although a high starting torque will be achieved, there will be lower efficiency. Also, there will be speed variation under operating conditions.

A squirrel cage induction motor has low starting torque. But, it has high efficiency and constant speed. Therefore, it is better to start a squirrel cage induction motor off load.

Also, the starting current in a squirrel cage induction motor can be very high; usually 4 to 5 times the normal full load current

The wound rotor induction motor allows the use of slip-rings. The slip-rings allow for the addition of resistance to the rotor circuit externally. The high starting current experienced by the cage induction motor can be regulated.

In general, for three-phase induction motors, the power factor is usually between about 0.8 and 0.9 lagging, and the full load efficiency is usually about 80–90%.

STARTING METHODS FOR INDUCTION MOTORS

Squirrel-cage rotor

1. **Direct-on-line starting**

In this type of method, starting current is high and may cause interference with electrical supplies to other consumers.

2. **Auto transformer starting**

In this method, an auto transformer will be used to reduce the stator voltage, E_1. Hence, the starting current will also reduce. However, the starting torque is seriously reduced. so the voltage is reduced only sufficiently to give the required reduction of the starting current.

3. **Star-delta starting**

In this method, for starting, the connections to the stator phase winding are star-connected,

CHAPTER FIVE

ARMATURE REACTION IN A D.C MACHINE

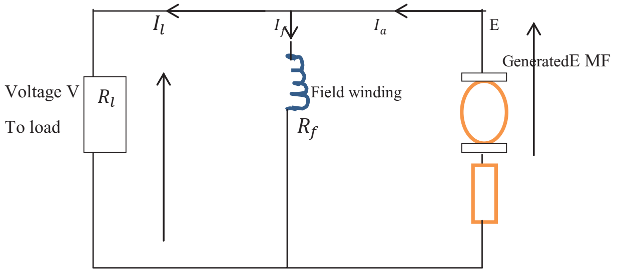


Figure 5.1 : Shunt wound generator

$I_a = I_f + I_l$

Terminal voltage V = E_g - $I_a R_a$ $\qquad$ $I_f = I_{sh}$

Then, terminal voltage V = E_g - (I_{sh}+ I_l) R_a

The figure above shows a shunt wound dc machine.

The field winding is connected in parallel with the armature winding. But if the field winding is connected in series with the armature winding, then we have a series wound machine as shown in the DC Series Generator below

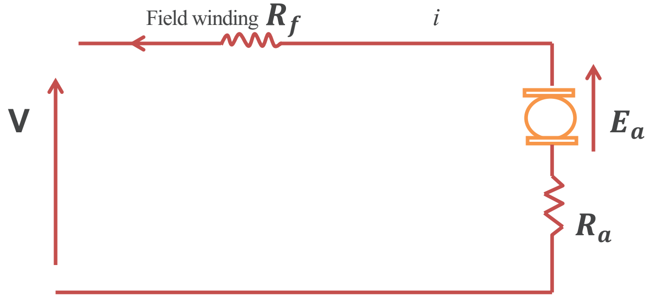


Figure 5.2 : DC Series Generator

Series-Wound Generator

A compound wound machine has both field winding and series winding. When a load is applied, each of these three machines behaves differently. The behavior of these dc machines under various conditions is shown by means of a graph called characteristics curve or simply characteristics.

In a motor, when a three phase supply is connected to the stator winding, a rotating magnetic field will be produced. The magnetic field flux will cut across the windings on the rotor and e.m.f will be induced on the rotor. This rotor magnetic field is called armature field or rotor magnetic field.

The armature or rotor magnetic field interacts with the main or stator magnetic field. The effect is that a force is produced. This force tends to turn the rotor in the same direction as the main rotating magnetic field. This same type of force will be applied on all the rotor conductors. Hence. Torque T in (Newton metre) will be produced and this will cause the rotor to rotate.

The effect that the armature magnetomotive force will have on the main field flux is called armature reaction.

The effect that the mmf (magnetomotive force) produced by armature current or the effect that the mmf (magnetomotive force) produced by armature winding will have on the main field flux produced by the field winding or stator winding is called armature reaction.

In a motor, armature reaction will increase the speed of the machine.

The total magnetic field produced in a motor is B_T

$$B_T = B_S + B_R$$

But in a generator, armature reaction will produce undesirable effects:

1. There will be reduction in the main field flux per pole
2. There will be distortion of the main field flux wave along the air gap
3. There will be reduction in the generated e.m.f
4. The distortion of main field flux will influence the limit of successful commutation

ARMATURE REACTION IN A D.C MACHINE

1. Armature reaction distorts the main field distribution in the air gap periphery.
2. It leads to demagnetization effect in the machine. Demagnetization effects of armature mmf will reduce the total flux per pole. This reduction is about 1 to 5% from no load to full load.

THE REMEDIES TO THE EFFECTS OF ARMATURE REACTION ARE AS FOLLOWS:

1. The use of compensating windings: The effects of armature reaction can be limited by using compensated windings. Compensated windings are the windings embedded in the slots cut in the pole phase of the D.C machine.
2. The use of strong main magnetic field flux: when the main field mmf is sufficiently strong. The effects of distortion will be less. The main field mmf must be greater than the full load armature mmf
3. The use of interpoles : inter poles are placed between the main poles: the effects of armature reaction can be overcome readily by using interpoles. Interpole winding is connected in series with the armature winding so that interpole mmf will neutralize the effects of armature reaction.
4. Reduction of armature flux: this is done by creating more reluctance in the path followed by the armature flux. But this must be done without reducing the the main field flux. We can do this by using field pole lamination with some rectangular holes punched in them as shown below

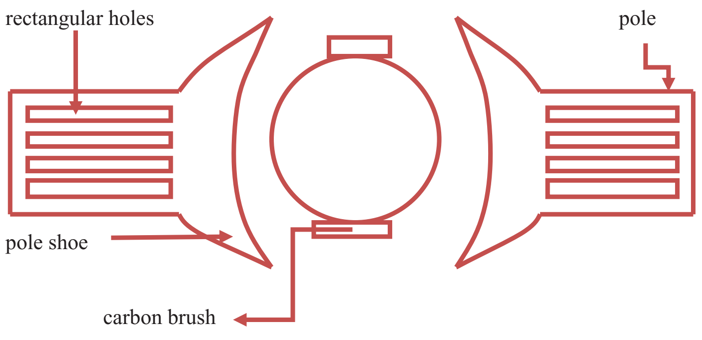


Figure 5.3 : Two pole D.C machine with punched field pole rectangular winding

The reluctance offered by using the field pole lamination above to the armature flux will be increased. The reluctance is offered due to the four air gap rectangular opening

5. The effects of armature reaction can also be limited by using high reluctance pole tips: whenever the reluctance of the pole tips is increased, the magnitude of the armature cross flux is reduced. The distortion of the reluctant flux density will be reduced as well. This is done by using chamfered pole as shown in the diagram below:

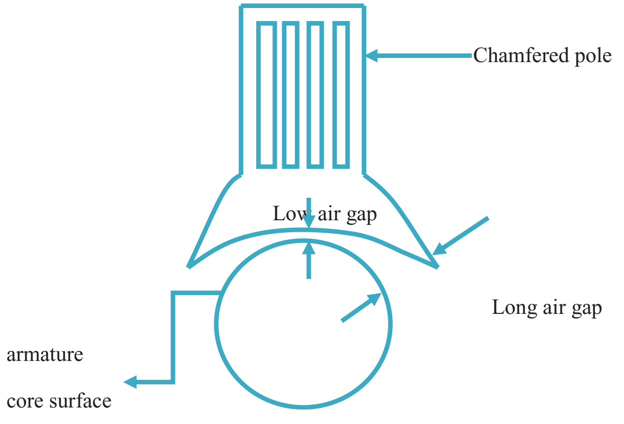


Figure 5.4 : Reduction of armature reaction by using chamfered pole

Example 9:

A 4 pole generator has a wave –wound armature with 780 conductors. It delivers 98 A on full load and the field current is 2A. The brush lead is 16^0 electrical, calculate

i. The armature demagnetization AT_d/ pole
ii. Cross magnetizing ampere turns per pole AT_C/ pole

Solution

i. The armature demagnetization = AT_d/ pole

$$= \frac{Z \quad I_a \quad \theta mechanical}{C \;\; X \;\; 360^0}$$

Where

1. Z = number of conductors
2. I_a= armature current which is = $I_{Load\ current} + I_{field} = 98 + 2$ = 100 A
3. $C = number\ of\ parallel\ paths$ which is =2 because the generator has a wave –wound armature
4. One complete electrical revolution is equal to half mechanical revolution

360^0 Complete electrical revolution is equal to 180^0 mechanical revolution

Since 360 = 2 x 180

This implies that $\theta electrical = 2\ x\ \theta mechanical$

In a 4 pole machine, P= number of pole , hence, P = 4

Since $\frac{4}{2} = \frac{P}{2} = 2$

Then, $\theta electrical = 2\ x\ \theta mechanical$ will become

$$\theta electrical = \frac{P}{2}\ x\ \theta mechanical$$

$\theta mechanical = \frac{2}{P}\ x\ \theta electrical$ since $\theta electrical = 16$

$$\theta mechanical = \frac{2}{4}\ x\ 16$$

$= 8^0$

5. Therefore,

The armature cross magnetizing ampere turn/pole $= AT_C/$ pole

$$= \frac{Z \quad I_a \quad \theta mechanical}{C \ X \ 360^0}$$

$$= \frac{780 \ x \ 100 \ x \ 8}{2 \ X \ 360^0}$$

$$= \frac{780 \ x \ 100 \ x \ 8}{2 \ X \ 360^0}$$

$= 866.67 \ AT_d/$ pole

or

The armature demagnetization $= AT_d/$ pole = back e.m.f turns/pole

$$= \frac{4 \ \theta electrical}{360^0} \text{ X } \frac{I_a \quad Z}{C \ X \ 2P}$$

$$= \frac{4 \ X \ 16}{360^0} \text{ X } \frac{100 \ x \ \ 780}{2 \ X \ 2 \ x \ 4}$$

$$= \frac{4992000}{5760}$$

$= 866.67 \ AT_d/$ pole

6. The armature cross magnetization $= AT_C/$ pole

$$= \frac{Z \quad I_a}{C} \ X \left[\frac{1}{2P} - \frac{\theta mechanical}{360}\right]$$

No of pole =4

$$= \frac{780 \;X\; 100}{2} \; X \left[\frac{1}{2\,X4} - \frac{8}{360}\right]$$

$$= \frac{780 \;X\; 100}{2} \; X \left[\frac{1}{2\,X4} - \frac{8}{360}\right]$$

= 39000 (0.1028)

=4008.3 AT_C/ pole or

The armature cross magnetization = AT_C/ pole

$$= \frac{180-2\,\theta electrical}{180} \;\text{X}\; \frac{I_a \;\; Z}{C \;\; X \;\; 2P}$$

$$= \frac{180-2\,X16}{180} \;\text{X}\; \frac{100\,X\,780}{2 \;\; X \;\; 2\,X4}$$

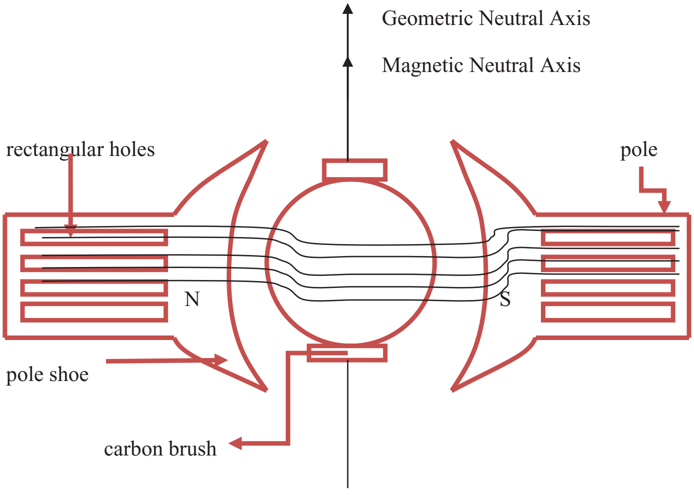
$$= \frac{148}{180} \;\text{X}\; \frac{78{,}000}{2 \;\; X \;\; 2\,X4}$$

$$= \frac{11544000}{2880}$$

= 4008.3 AT_C/ pole

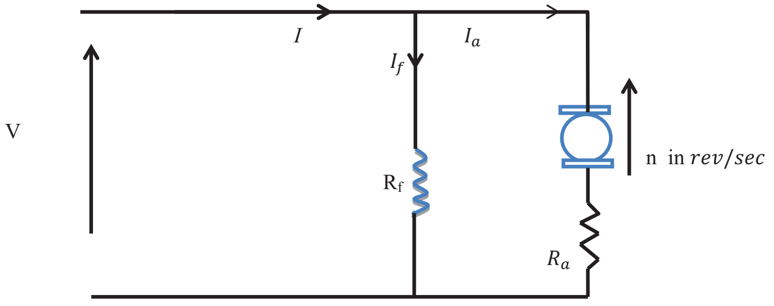
CROSS MAGNETISM IN ELECTRICAL MACHINES

The figure below shows the flux direction for main pole of a 2 pole dc machine when there is no load connected i.e. at no load.



The main field flux produced by the field winding = mmf. In a motor, shown below:

Figure 5.5 : main field flux produced by the field winding



At no load, $I_a = 0$

Figure 5.6 : Shunt wound motor

Hence, current I will produce magnetic field at the field winding.

The main field flux produced by the field winding = mmf

$$\text{This mmf} = \text{F} = I_f N_f = \text{H}\, l$$

$Magnetic\ flux\ density = B = \mu H$

Reluctance in the air gap = S= $\frac{F}{\varnothing}$

The magnetic flux produced by the field winding at the no load stage = $\varnothing_f$ *and it* will be represented by horizontal line in the figure below

On load, i.e. when the machine is loaded, current will flow into the armature winding and the diagram above will become

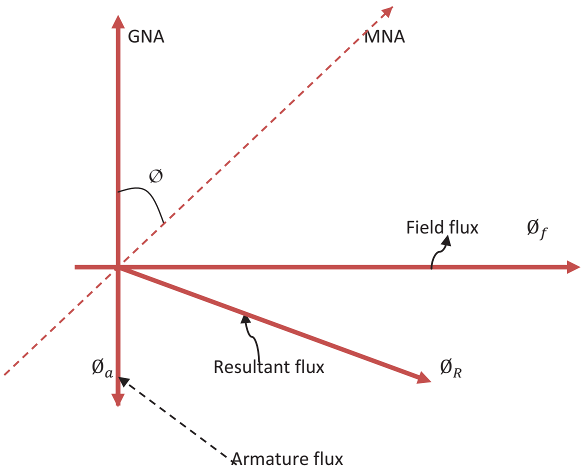


Figure 5.7 Geometric Neutral Axis against the field flux

You will observe that the path of the armature flux is perpendicular to the direction of the main field flux. Therefore the path of the armature flux is said to cross the path of the main field flux. Finally, the effect of the armature flux on the main field is totally cross magnetizing. The flux created by the armature mmf is therefore called cross flux or cross magnetizing flux as shown below

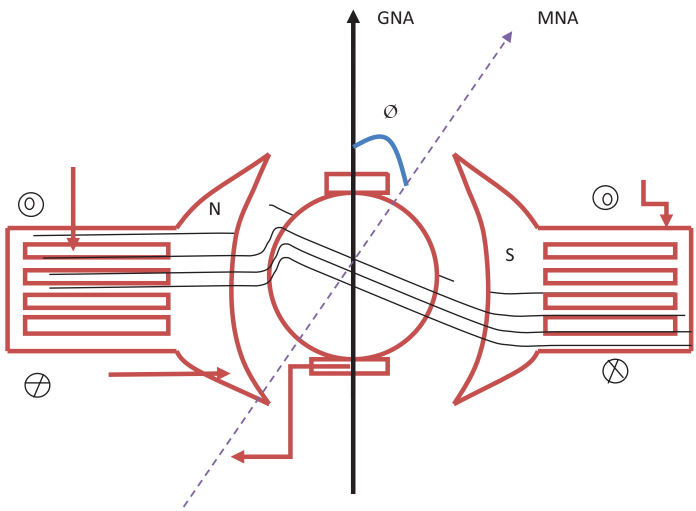


Figure 5.8 : Cross flux or cross magnetizing flux

The main field flux produced by the field winding = mmf =F

N = Number of turns

I = current

H = Magnetizing force

S = relunctance

B = magnetic flux density

Ø = magnetic flux

A = area

µ = absolute permittivity

$l = lenght$
μ_o = permitivity of free space = magnetic space constant for air or any non magnetic material = $4 \times \pi \times 10^{-7}$ H/m
$\mu_r = relative\ permitivity$

$$\mu_r = \frac{flux\ density\ in\ material}{flux\ density\ in\ vacuum}$$

From the equation above, relative permittivity of free space for vacuum = 1

This mmf $= F = NI = Hl$

Magnetic flux density $= B = \mu H$ *Also,* $\O = BA$

Reluctance in the air gap = S= $\frac{F}{\O}$

$$S = \frac{I.N.}{BA} = \frac{Hl}{BA}$$

$$S = \frac{HL}{BA}$$

$$S = \frac{HL}{\mu H\ A} = \frac{l}{\mu\ A}$$

Since, $\mu = \mu_o \mu_r$

$$S = \frac{Hl}{\mu H\ A} = \frac{l}{\mu_o \mu_r\ A}$$

Remember that Inductance $L = \frac{N^2}{S}$ and energy in an inductor is = $E = \frac{1}{2} LI^2$

CLASS WORK

State the meaning of commutation and excitation in a dc machine

1. The reversal of current in the armature coil by means of brush and commutator bars is called commutator process.

It is expected that good commutation

1. Will produce no sparking at the brushes
2. The commutator surface will remain unaffected during the continuous operation of the dc machine
3. The commutator surface will not get damaged

When these conditions are not met then there is poor commutation.

2. Excitation

Excitation is the process of producing magnetic field by means of electric current in an electrical machine.

3. State three factors to be considered in the choice of a generator
   i. Output power of the generator in kW
   ii. Apparent power of the generator in kVA
   iii. Total load ratings : the total active power of the generator must be greater than the load ratings
4. Mention two reasons why a running generator will not produce output power
   i. If the carbon brushes contain particles, dust or other elements that prevent good contact with the anode and cathode parts of the commutator
   ii. If the circuit breaker is not working or if it is off
   iii. If the oil chamber is empty: starting the generator in this case might destroy the generator completely
   iv. If the switch is not working
   v. If the ignition coil is bad
   vi. If the anode and cathode parts of the sparking plug had been in contact or if the sparking plug is bad

OPERATING CHARACTERISTICS AND USES OF SERIES, SHUNT AND CUMULATIVE COMPOUND MOTORS

CHARACTERISTICS OF SERIES MOTOR

1. **Series motor has high starting torque**
2. **It has variable speed**
3. It has good accelerating torque

USES OF SERIES MOTORS

1. **It is used in conveyors**
2. It is used for traction work i.e. electric locomotive
3. It is used for rapid transit system
4. It is used for trolleys
5. It is used for cranes and hoist

CHARACTERISTICS OF SHUNT MOTORS

1. **The speed is approximately constant**
2. It has medium starting torque

USES OF SHUNT MOTORS

1. It is used in reciprocating pump
2. It is used in centrifugal pump
3. For machine tools
4. It is used in the operation of fans and blowers
5.

CHARACTERISTICS OF CUMMULATIVE COMPOUND MOTORS

1. It has variable speed
2. It has adjustable varying speed

USES OF CUMMULATIVE COMPOUND MOTORS

1. It is used in elevators
2. It is used in conveyors
3. It is used in heavy planers
4. It is used in printing press
5. It is used in air compressor
6. It is used in ice machine
7. It is used in rolling mills

EXAMPLE 10

An electric magnetic relay has an exciting coil of 800 turns. The coil has a cross sectional area of 5cm x 5cm. Neglecting the reluctance of the magnetic field circuit and cringing,

1. Find the coil inductance if the air gap length is 0.6 cm
2. Find the field energy stored for a coil current of 1.5 A

SOLUTION

Since,

Reluctance in the air gap = S= $\frac{F}{Ø}$

$$S= \frac{I.N.}{BA} = \frac{Hl}{BA}$$

$$S= \frac{HL}{BA}$$

$$S= \frac{HL}{\mu H\ A} = \frac{l}{\mu\ A}$$

Since, $\mu = \mu_o \mu_r$

$$S= \frac{Hl}{\mu H\ A} = \frac{l}{\mu_o \mu_r\ A}$$

$$S = \frac{0.6 \, x\, 10^{-2}}{4\,x\,\pi\,x\,10^{-7}\,x\,1x\,5\,x\,10^{-2}\ X\,5\,x\,10^{-2}}$$

$$S = \frac{0.6\,x\,10^{+7+2+2-2}}{4\,x\,\pi\,x\ x\,1x\,25}$$

$$S = \frac{0.6\,x\,10^{+9}}{4\,x\,\pi\,x\ 1\,x\,25}$$

$$S = \frac{600000000}{4\,x\,\pi\,x\ x\,1x\,25}$$

S = 1909611.712 AT/wb

Since

Inductance $L = \frac{N^2}{S}$

$$L = \frac{800^2}{1909611.712}$$

L = 0.335 Henry

Energy in an inductor is = $E = \frac{1}{2} LI^2$

$$E = \frac{1}{2}\ 0.335 \text{ x } 1.5^{\ 2}$$

E = 0.377 joule

Example 11

A 4 pole series motor has 1200 wave connected armature conductors. At certain load, the flux per pole is 36.8 mWb. The total mechanical torque is 218 Nm. Calculate the line current taken by the motor and the speed at which it will run with an applied voltage of 480 V.

Total motor resistance is three ohms (3 ohms)

SOLUTION

a. We have been able to prove that

$$T = \frac{\text{Mechanical Power P}}{2\pi n} = \frac{EI_a}{2\pi n} \qquad \text{and } E = \frac{2p\emptyset nZ}{C}$$

From the above 2 equations

$$I_a = \frac{T\ X\ 2\pi n}{E} \quad \frac{T\ X\ 2\pi n}{\frac{2p\emptyset nZ}{C}}$$

$$I_a = \frac{T\ X\ 2\pi n}{\frac{2p\emptyset nZ}{C}}$$

$$I_a = \frac{218\ \ X\ 2\ X\ 3.142\ X\ n}{\frac{2\ X 2\ X 0.0368\ X\ \text{n X 1200}}{2}}$$

$$I_a = \frac{218\ \ X\ 2\ X\ 3.142\ X\ n}{\frac{2\ X 2\ X\ 0.0368\ X\ \text{n X 1200}}{2}}$$

$$I_a = \frac{1369.912}{88.320}$$

$I_a = 15.51\ A$

a. Since we know that

$$E = \frac{2p\text{Ø}nZ}{C}$$

Then,

$$n = \frac{E\ X\ c}{2p\text{Ø}Z}$$

Also, we have seen that in an electric motor, E = V - $I_a R_a$

E = 480 - 15.51 x 3

E = 433.47 V

From, $$E = \frac{2p\text{Ø}nZ}{C}$$

$$n = \frac{E\ X\ c}{2p\text{Ø}Z}$$

The speed at which it will run with an applied voltage of 480 V = $\frac{433.47\ X\ 2}{2\ X 2\ X\ 0.0368\ X\ 1200}$

The speed at which it will run with an applied voltage of 480 V = $\frac{433.47\ X\ 2}{2\ X 2\ X\ 0.0368\ X\ 1200}$

The speed at which it will run with an applied voltage of 480 V = $\frac{866.94}{176.64}$

= 4.908 rev/sec

= 294.477 rev/min

EXAMPLE 12

The following data pertain to the magnetization curve or open circuit characteristics or no load characteristics of a dc shunt generator at 1500rpm

$I_{f\ amps}$	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
$E_a\ volts$	6	60	120	172	205	225	230	238	240

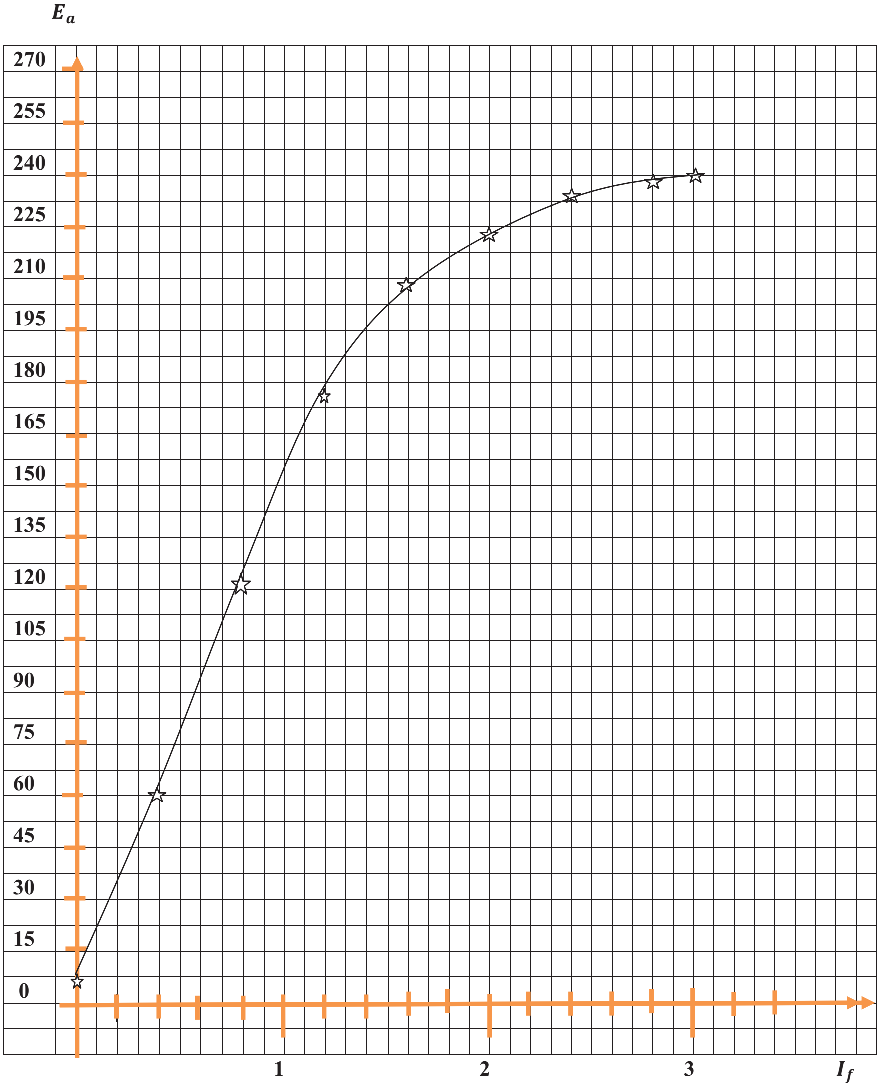
For this generator, draw the no load characteristics curve and show the field resistance line Hence, obtain

1. The voltage on open circuit to which the machine will be built up (the no load e.m.f) for a total shunt field resistance of 100 Ω
2. The critical value of the shunt field resistance at 1500 rpm
3. The critical speed for the shunt field resistance of 100 Ω
4. The magnetization curve at 1200 rpm and therefore the open circuit voltage for field resistance of 100 Ω
5. The terminal voltage of the generator if the armature resistance is 0.3 Ω, armature current is 50 A, and the speed is 1500rpm. Neglect the armature reaction

Solution

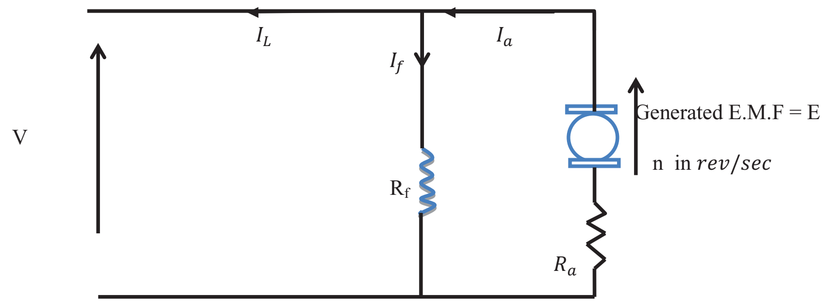
The magnetization curve is the graphical representation of the generated voltage E_a and the field current I_f . *it is presented in the characteristics graph below*:

I_f amps	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
E_a volts	6	60	120	172	205	225	230	238	240



Now let us draw the field resistance line

From the circuit diagram of a shunt wound generator below



V = $I_f \, x \, R_f$

The field resistance line is drawn by choosing a particular high terminal voltage = V. Let us choose 240V

Then, if we choose V to be = 240 V and the question says shunt field resistance = R_f is 100 Ω

240 = $I_f \, x \, 100$

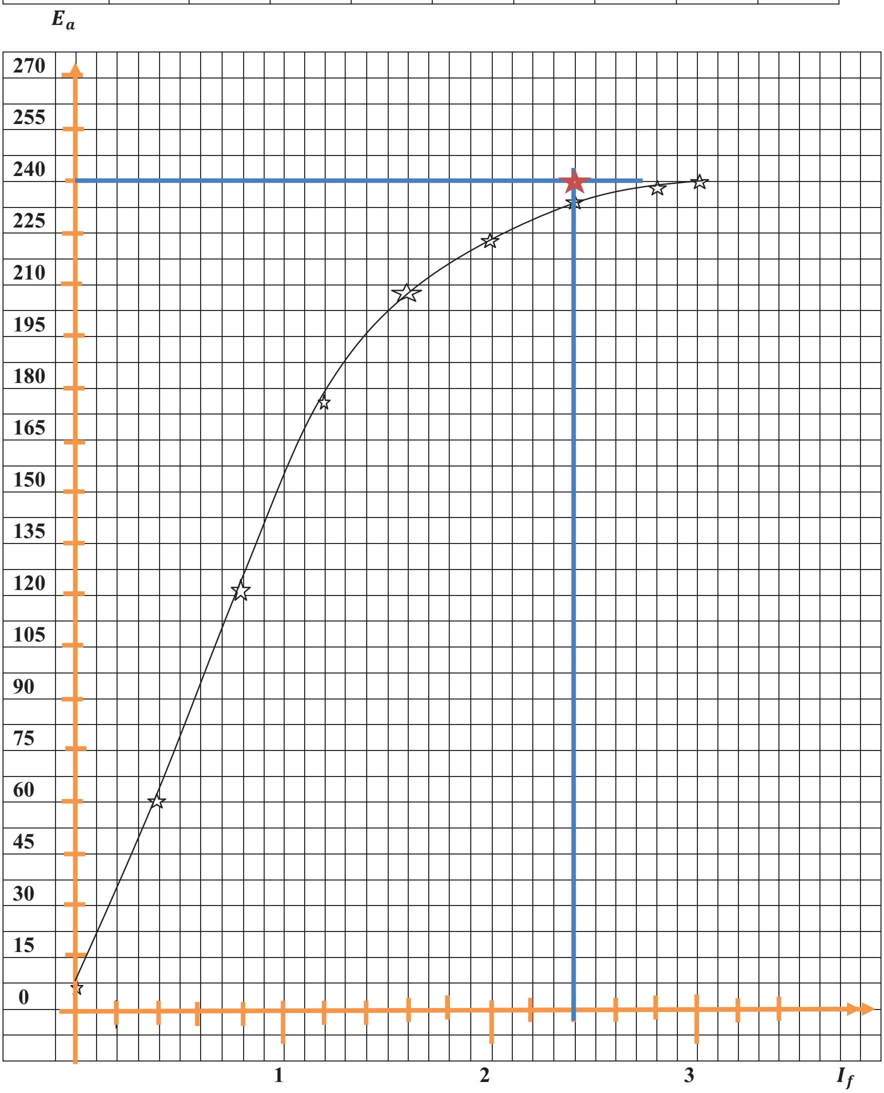
Therefore $I_f = 2.4 \, A$

Therefore for field resistance, we have the table below:

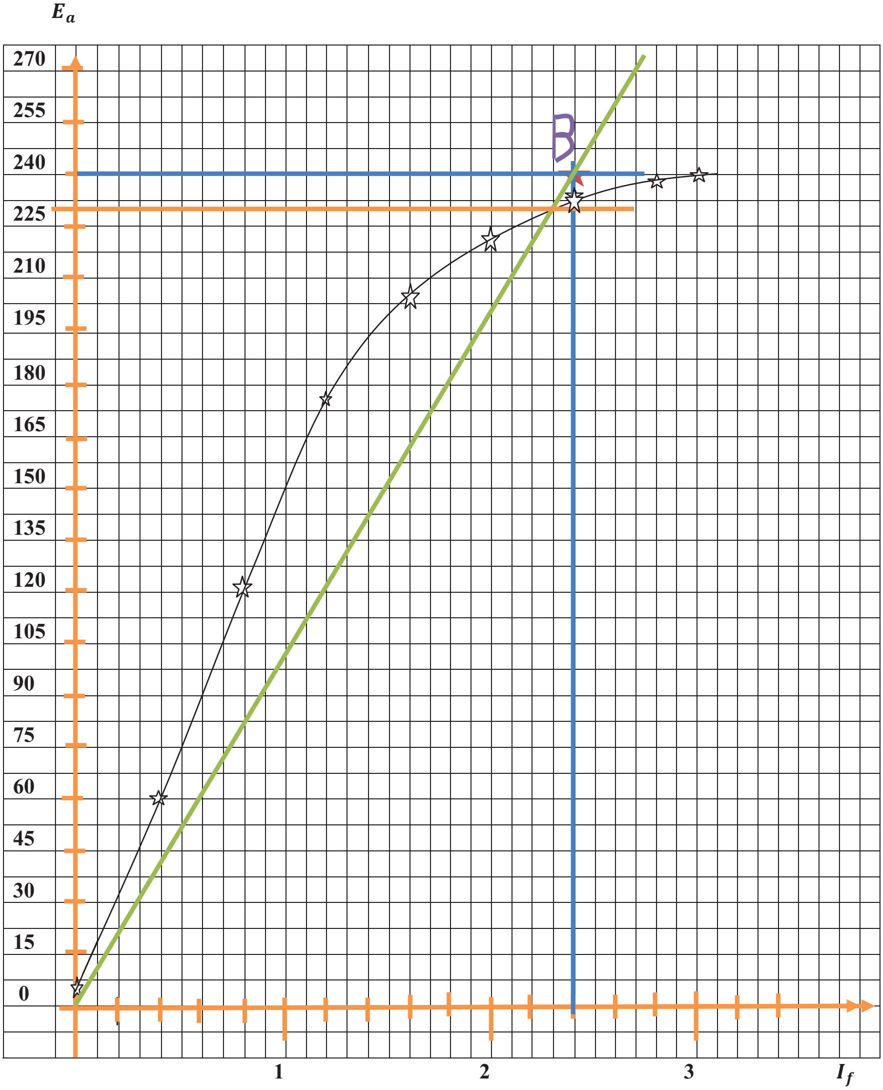
	Terminal voltage =V	Field resistance = R_f	Field Current = I_f
	240 V	100 Ω	2.4 A

Plot the field resistance line using (240V against 2.4 A) from the origin

I_f *amps*	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
E_a *volts*	6	60	120	172	205	225	230	238	240



$I_{f\ amps}$	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
E_a volts	6	60	120	172	205	225	230	238	240



You will observe that the field resistance meets the open circuit characteristics curve at point B. just trace that point to the E_a axis. That will be the no load e.m.f required

E_a = no load e.m.f = 230 V

CRITICAL SPEED

This is the speed at which the d.c shunt generator above will just fail to build up voltage with no external resistance in the field circuit. During this speed, the generator is running but there will be no voltage build up at no load.

The cause of this problem may be as a result of any of the following factors:

1. When the speed of the generator is less than the critical speed: if the armature speed is less than the critical speed, then voltage build up process will not take place.
   This is the solution
   Increase the speed of the prime mover above the critical speed
2. When there is no residual magnetism: in the magnetic circuit of a generator, voltage build up begins only is there is residual magnetism. This occurs when a machine has lost its residual magnetism due to aging or long use
   This is the solution
   a. **Excite the field winding from a separate dc source for a few second with armature at rest**
   b. **Disconnect the d.c source**
   c. **By doing so, the main pole will possess residual flux**
   d. **The voltage build up process can now take place**
3. When there is high field circuit resistance there will not be voltage build up process: if the field resistance is more than the critical field resistance, voltage will not build up. The voltmeter connected in parallel to the armature terminal will only read a low voltage.
   These are the possible causes
   a. **dirty commutator**
   b. **large external resistance in the field circuit**
   c. **open circuit in the field or armature connection**

These are the remedies

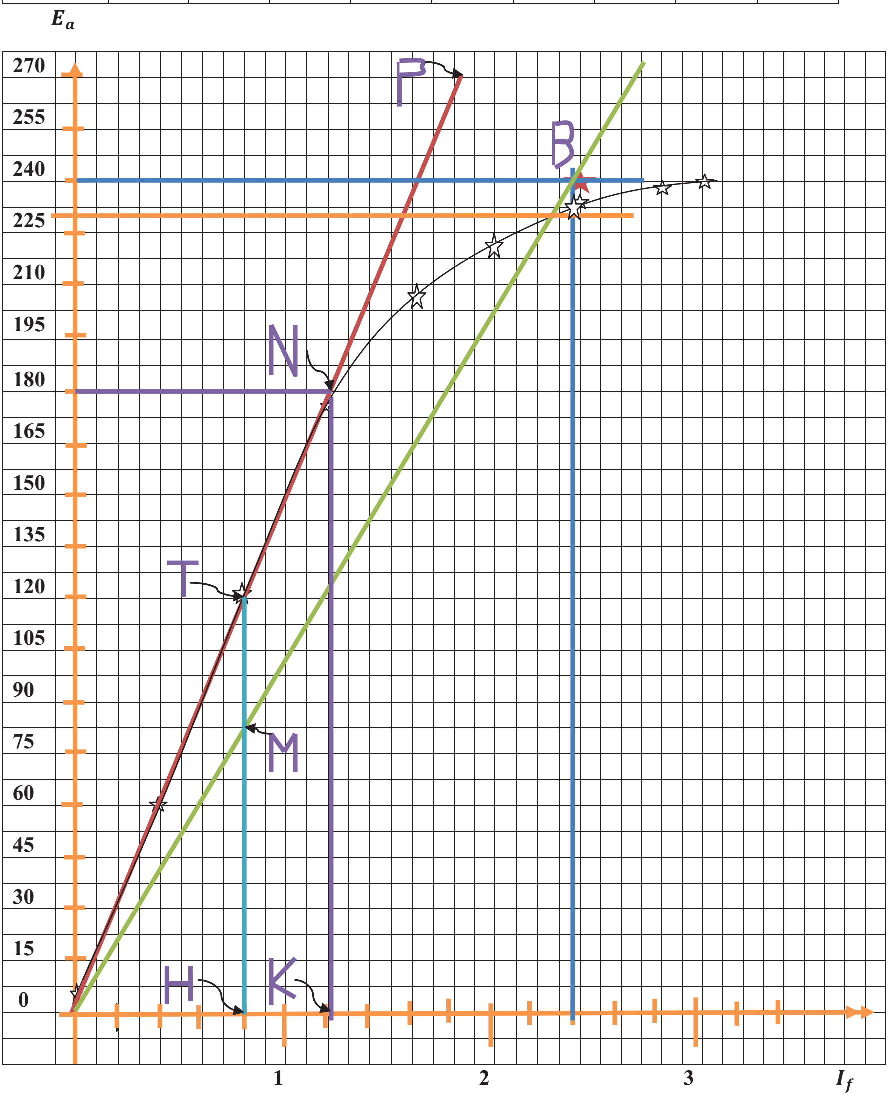
a. **clean the commutator surface and ensure good contact between the commutator surface and brushes**
b. **adjust the external rheostat to zero ohms**
c. **check the field or armature winding whether it is open**

4. Field connection reversal can also lead to the problem of no voltage build up:
   The small voltage from the residual magnetism should circulate current in the field coils in a direction that will produce flux lines aiding the residual flux. But when the field connection is not correct, the flux produced by the small field current will oppose the residual flux. Therefore, the generated voltage will decrease to zero

   This is the solution
   Reverse the field connection with respect to the terminals of the armature

b. Now let us calculate The critical value of the shunt field resistance at 1500 rpm and The critical speed for the shunt field resistance of 100 Ω
   i. Draw a line OP below it should pass through the origin and tangential to the magnetization curve at 1500 rpm
   ii. Calculate the slope of line OP. The slope of the line is the critical value of the shunt field resistance

I_f amps	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
E_a volts	6	60	120	172	205	225	230	238	240



Slope of line OP = $\frac{180}{1.21}$

Slope of line OP = 148.76 Ω

This is the critical resistance of the shunt field at 1500 rpm

c. To obtain the critical speed
   i. Choose any suitable point T on the straight line portion of the magnetization curve or open circuit characteristic
      iii. Draw a vertical line from T to meet the field resistance line at M
         iii. Extend the line in ii above to meet the horizontal axis at point H

From the vertical line described above, the critical speed = n_C can be obtained

Where, critical speed = $n_C = \frac{HM}{HT}$ X generator speed

Critical speed = $n_C = \frac{82.5}{120}$ X 1500 rpm

Critical speed = $n_C = \frac{123750}{120}$

Critical speed = n_C = 1031.25 rpm

Or just apply $E = k\,Ø\,n$

So that E_1 = k Ø. n_1 *From the open circuit characteristic curve* E_1 = 120 V, $n_1 = 1500rpm$

120 = k Ø. 1500 ……………………..(1)

Also, E_2 = k Ø. n_C *From the open circuit characteristic curve* E_2 = 82.5 V, *critical speed* = n_2 *and it is unknown*

82.5 = k Ø. n_C ………………………..(2)

Dividing equation 1 by 2, we will have

$$\frac{120}{82.5} = \frac{k\,Ø.\,1500}{k\,Ø.\,n_C}$$

Critical speed = $n_C = \frac{82.5}{120}$ X 1500 rpm

$$n_C = \frac{123750}{120}$$

Critical speed = n_C = 1031.25 rpm

d. Now, let us obtain the magnetization curve at 1200 rpm and therefore the open circuit voltage for field resistance of 100 Ω

We are going to obtain another set of values for the generated e.m.f = E_a

using $E = k\, Ø\, n$

The table below is for the field current and generated e.m.f at a speed at 1500rpm

I_f *amps*	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
$E_1 = E_a$ *volts*	6	60	120	172	205	225	230	238	240

i. *From* $E_1 = k\, Ø_{.}\, n_1$ ....(1)

$E_2 = k\, Ø_{.}\, n_2$ ....(2)

Dividing the two equations above, we have

$$\frac{E_1}{E_2} = \frac{k\, Ø_{.}\, n_1}{k\, Ø_{.}\, n_2}$$

$$E_2 = \frac{E_1\, k\, Ø_{.}\, n_2}{k\, Ø_{.}\, n_1}$$

$$E_2 = \frac{E_1\; n_2}{n_1}$$

E_1 = 6, 60, 120, 172, 202, 225, 230,238, 240 $\quad n_1 = 1500rpm,\quad$ E_2 are unknown values

$$n_2 = 1200rpm$$

Therefore,

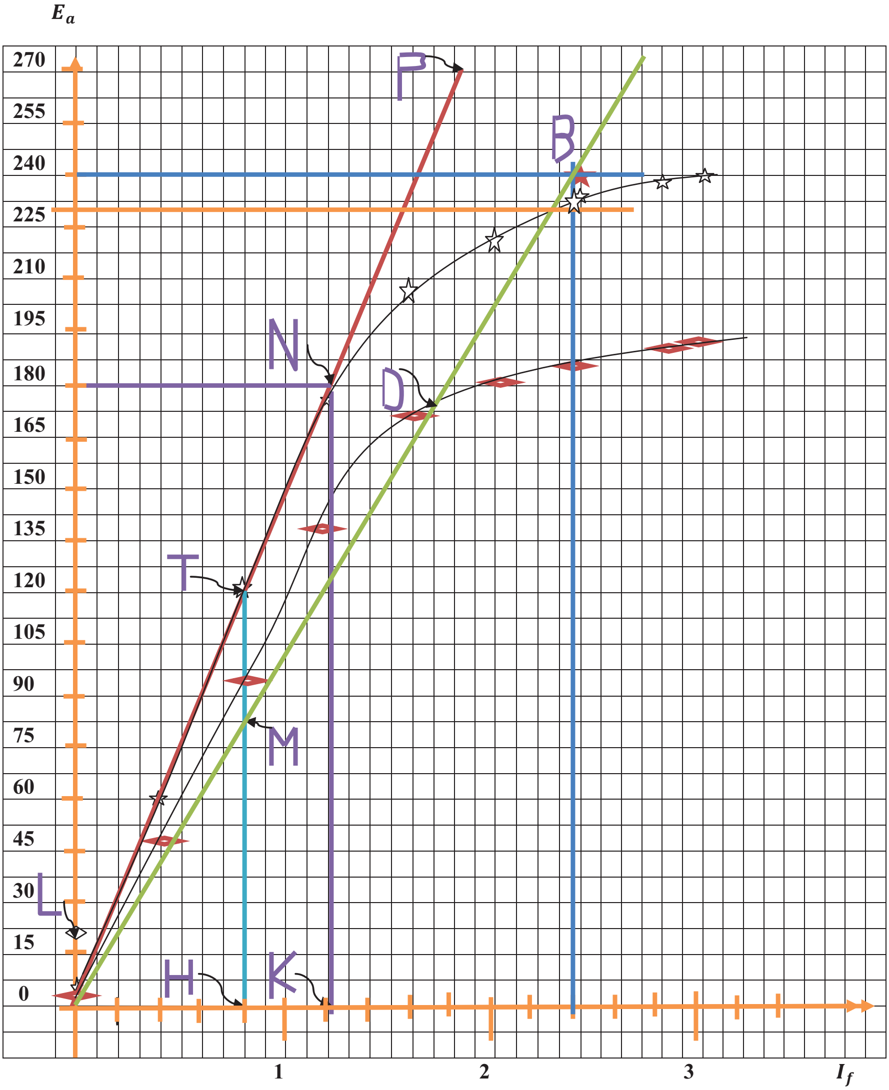
We obtain the table below

The table below is for the field current and generated e.m.f at a speed at 1500rpm

$I_{f\ amps}$	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
$E_1 = E_a\ volts$	6	60	120	172	205	225	230	238	240
E_2	$\frac{E_1\ n_2}{n_1}$	$\frac{E_1\ n_2}{n_1}$	$\frac{E_1\ n_2}{n_1}$	$\frac{E_1\ n_2}{n_1}$	$\frac{E_1\ n_2}{n_1}$	$\frac{E_1\ n_2}{n_1}$	$\frac{E_1\ n_2}{n_1}$	$\frac{E_1\ n_2}{n_1}$	$\frac{E_1\ n_2}{n_1}$
E_2	$\frac{6x1200}{1500}$	$\frac{60x1200}{1500}$	$\frac{120x1200}{1500}$	$\frac{172x1200}{1500}$	$\frac{205x1200}{1500}$	$\frac{225x1200}{1500}$	$\frac{230x1200}{1500}$	$\frac{238x1200}{1500}$	$\frac{240x1200}{1500}$
E_2	4.8	48	96	137.6	164	180	184	190.4	192

Then we can easily obtain the magnetization curve at 1200rpm as shown below

I_{f} amps	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
$E_a = E_2\,V$	4.8	48	96	137.6	164	180	184	190.4	192



When the speed is now 1200 rpm

V = $I_f \ x \ R_f$

The field resistance line is drawn by choosing a particular high terminal voltage = V. Let us choose 200V

Then, if we choose V to be = 240 V and the question says shunt field resistance = R_f is 100 Ω

200 = $I_f \ x \ 100$

Therefore $I_f = 2 \ A$

Therefore for field resistance, we have the table below:

	Terminal voltage =V	Field resistance = R_f	Field Current = I_f
	200 V	100 Ω	2 A

Plot the field resistance line using (200V against 2.4 A) from the origin

the field resistance line using (200V against 2.4 A) from the origin had been plotted already

when the speed of the generator is now 1200 rpm, the point of intersection of the field resistance and the new magnetization curve gives the no load E.M.F at 1200 rpm

From the no load characteristics curve,

The open circuit voltage or no load E.M.F at 1200 rpm is at point D and it is = 175 V

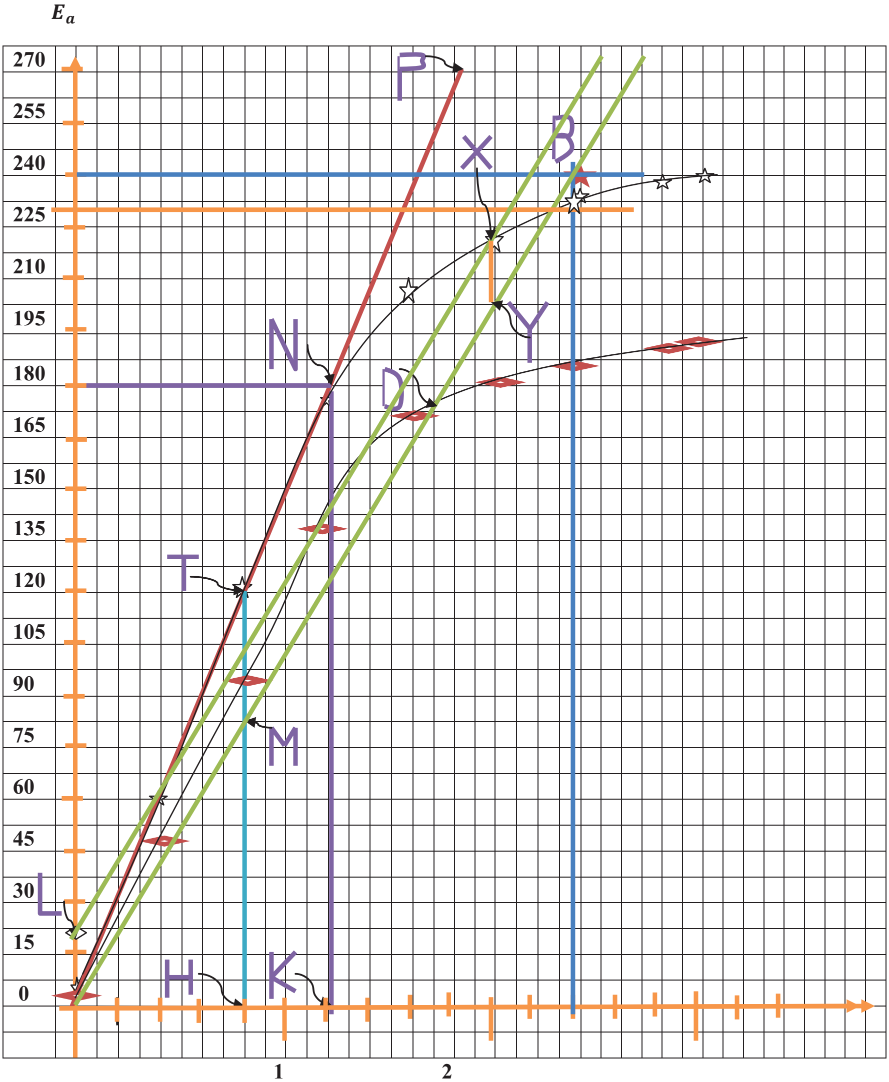
e. Finally we shall calculate the terminal voltage of the generator if the armature resistance is 0.4 Ω, armature current is 50 A, and the speed is 1500rpm. Neglect the armature reaction

From $E = V + I_a R_a$

Since, $I_a = 50 \ A$
$R_a = 0.4 \ \Omega$

Then, $I_a R_a$ = 50 x 0.4 = 20 V
Now return to the no load characteristics curve.

I_f amps	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
$E_a = E_2 V$	4.8	48	96	137.6	164	180	184	190.4	192



From the origin, along the $\boldsymbol{E_a}$ **axis i.e. along the vertical axis**

The voltage loss is= $I_aR_a = $ 20V, and this corresponds to point L on the curve

Now draw line LX parallel to the field resistance line of 100 Ω.

X is a point on the magnetization curve when the speed is 1500rpm

From X, draw a vertical line to meet the field resistance line at point Y. This point Y corresponds to the Terminal voltage V when the voltage drop is 20 V.

From the open circuit characteristics curve, terminal voltage is value of E_a at point Y

Hence, the terminal voltage when the speed is 1500rpm and the voltage drop is = 20 V

And

$$E = V + I_aR_a$$

$$.E = 202.5 + 20$$

The generated E.M.F when the speed is 1500rpm = E = 222.5

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